6-1

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Study Guide

Divisibility Patterns

The following rules will help you determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

A number is divisible by:

- 2 if the ones digit is divisible by 2.
- 3 if the sum of the digits is divisible by 3.
- 4 if the number formed by the last two digits is divisible by 4.
- 5 if the ones digit is 0 or 5.
- 6 if the number is divisible by 2 and 3.
- 8 if the number formed by the last three digits is divisible by 8.
- 9 if the sum of the digits is divisible by 9.
- 10 if the ones digit is 0.

Example **Determine whether 2,120 is divisible by 2, 3, 4, 5, 6, 9, or 10.**

2: The ones digit is divisible by 2.
2,120 is divisible by 2.

3: The sum of the digits $2 + 1 + 2 + 0 = 5$, is not divisible by 3.
2,120 is not divisible by 3.

4: The number formed by the last two digits, 20, is divisible by 4.
2,120 is divisible by 4.

5: The ones digit is 0.
2,120 is divisible by 5.

6: The number is divisible by 2 but not by 3.
2,120 is not divisible by 6.

8: The number formed by the last 3 digits, 120, is divisible by 8.
2,120 is divisible by 8.

9: The sum of the digits, $2 + 1 + 2 + 0 = 5$, is not divisible by 9.
2,120 is not divisible by 9.

10: The ones digit is 0.
2,120 is divisible by 10.
2,120 is divisible by 2, 4, 5, 8, and 10.

Determine whether the first number is divisible by the second number. Write yes or no.

1. 4,829; 9	**2.** 482; 2	**3.** 1,692; 6
4. 1,355; 10	**5.** 633; 3	**6.** 724; 4
7. 3,714; 8	**8.** 912; 9	**9.** 559; 5
10. 20,454; 6	**11.** 616; 8	**12.** 3,000; 4

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6-2 Study Guide

Prime Factorization

A whole number greater than 1 with exactly two factors, 1 and itself, is called a **prime number.**

Example 1 19 is a prime number. It has only 1 and 19 as factors.

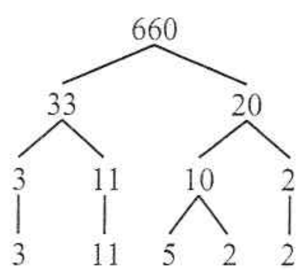
A whole number greater than 1 with more than two factors is called a **composite number.**

Example 2 18 is a composite number. It has 1, 2, 3, 6, 9, and 18 as factors.

The numbers 0 and 1 are neither prime nor composite.

A composite number may be written as the product of prime numbers. This product is the **prime factorization** of the number.

Example 3 **Find the prime factorization of 660.**



Write the number as the product of two factors.

Continue to factor until only prime factors remain.

The prime factorization of 660 is $3 \cdot 11 \cdot 5 \cdot 2 \cdot 2$.

Determine whether each number is prime, composite, or neither.

1. 28 **2.** 47 **3.** 39 **4.** 61

5. 53 **6.** 0 **7.** 159 **8.** 1

Find the prime factorization of each number.

9. 30 **10.** 155 **11.** 169 **12.** 100

13. 86 **14.** 98 **15.** 495 **16.** 40

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6-3 Study Guide

Greatest Common Factor

The greatest of the factors common to two or more number is the greatest **common factor (GCF).** One way to find the GCF is to list all of the factors of each number. Then find the greatest number that is in both lists.

Example 1 **Find the GCF of 32 and 48.**

Factors of 32: 1, 2, 4, 8, 16, 32
Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

The common factors of 32 and 48 are 1, 2, 4, 8, and 16.
The greatest common factor of 32 and 48 is 16.

You can use prime factorization to find the greatest common factor.

Example 2 **Find the greatest common factor of 90 and 120.**

$$\begin{aligned} 90 &= 3 \cdot 30 \\ &= 3 \cdot 15 \cdot 2 \\ &= 3 \cdot 5 \cdot 3 \cdot 2 \end{aligned} \qquad \begin{aligned} 120 &= 3 \cdot 40 \\ &= 3 \cdot 10 \cdot 4 \\ &= 3 \cdot 2 \cdot 5 \cdot 2 \cdot 2 \end{aligned}$$

90 and 120 have 2, 3, and 5 as common factors.
The product of the common factors is the greatest common factor.
$2 \cdot 3 \cdot 5 = 30$
30 is the greatest common factor of 90 and 120.

Find the GCF of each set of numbers.

1. 24, 40	**2.** 18, 30	**3.** 12, 48
4. 36, 90	**5.** 50, 20	**6.** 15, 17
7. 52, 16	**8.** 63, 42	**9.** 25, 40
10. 36, 12, 72	**11.** 28, 49, 105	**12.** 30, 45, 75

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6-4 Study Guide

Rational Numbers

Whole numbers are numbers in the set {0, 1, 2, 3, . . .}. **Integers** are the whole numbers and their opposites.

Rational numbers are numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples $6\frac{1}{7}$ can be written as $\frac{43}{7}$.

58 can be written as $\frac{58}{1}$.

0.65 can be written as $\frac{65}{100}$.

-79 can be written as $-\frac{79}{1}$.

-0.7 can be written as $-\frac{7}{10}$.

0 can be written as $\frac{0}{1}$.

When a rational number is expressed as a fraction, it is commonly written in simplest form. A fraction is in simplest form when the GCF of the numerator and denominator is 1.

Example **Write $\frac{18}{24}$ in simplest form.**

Method 1 Divide by the GCF.

$18 = 2 \cdot 3 \cdot 3$
$24 = 2 \cdot 2 \cdot 2 \cdot 3$

The GCF is $2 \cdot 3 = 6$.

$$\frac{18}{24} \xrightarrow{\div 6} \frac{3}{4}$$

Method 2 Use prime factorization.

$$\frac{18}{24} = \frac{\cancel{2} \cdot \cancel{3} \cdot 3}{\cancel{2} \cdot 2 \cdot 2 \cdot \cancel{3}} = \frac{3}{4}$$

The slashes show that the numerator and denominator are divided by $2 \cdot 3$, the GCF.

Since the GCF of 3 and 4 is 1, the fraction $\frac{3}{4}$ is in simplest form.

Name all sets of numbers to which each number belongs.

1. $4\frac{1}{2}$
2. 14.5
3. 17
4. 0.78

Write each fraction in simplest form.

5. $\frac{16}{26}$
6. $\frac{24}{72}$
7. $\frac{36}{78}$
8. $\frac{21}{56}$
9. $\frac{30}{75}$
10. $\frac{20}{48}$
11. $\frac{45}{81}$
12. $\frac{28}{49}$

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6-5 Study Guide

Rational Numbers and Decimals

To change a fraction to a decimal, divide the numerator by the denominator.

Example 1 Express $\frac{5}{8}$ as a decimal.

Use a calculator.

5 $\boxed{\div}$ 8 $\boxed{=}$ *0.625*

$\frac{5}{8} = 0.625$

The remainder is 0. 0.625 is a terminating decimal.

Use paper and pencil.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Annex zeros as needed.

$\frac{5}{8} = 0.625$

A terminating decimal can be written as a fraction with a denominator of 10, 100, 1000, and so on.

Examples Express each decimal as a fraction or mixed number in simplest form.

2 $0.52 = \frac{52}{100}$
$= \frac{13}{25}$

3 $-1.4375 = -1\frac{4375}{10{,}000}$
$= -1\frac{7}{16}$

Express each fraction as a decimal.

1. $\frac{3}{5}$ 2. $-\frac{7}{10}$ 3. $-\frac{7}{4}$ 4. $-\frac{15}{32}$ 5. $\frac{3}{20}$

6. $5\frac{7}{25}$ 7. $-\frac{24}{16}$ 8. $-6\frac{3}{8}$ 9. $\frac{14}{35}$ 10. $\frac{5}{40}$

Express each decimal as a fraction or mixed number in simplest form.

11. 0.08 12. −3.75 13. 0.015 14. −0.6 15. 1.05

16. −12.34 17. 2.1875 18. −0.875 19. −8.85 20. 9.5

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6-6 Study Guide

Integration: Probability
Simple Events

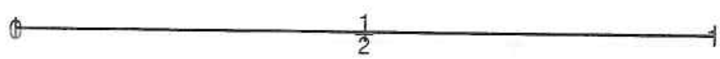
If you toss a coin, there are two possible outcomes, heads or tails. Each outcome has the same chance of occurring. The two outcomes are equally likely. A particular outcome, such as tossing a tail, or a set of outcomes is an event. **Probability** is the chance that an event will happen.

$$\text{Probability} = \frac{\text{number of ways an event can occur}}{\text{number of possible outcomes}}$$

Tossing tails can occur 1 way out of 2 possible outcomes, So, $P(\text{tail}) = \frac{1}{2}$.

The probability of an event can be written as a number from 0 to 1. If it is impossible for an event to happen, the event has a probability of 0. If an event is certain to happen, it has a probability of 1.

Impossible | Equal chance | Certain



Example **Maryanne writes the letters in her name on tennis balls, one letter per ball. If she puts the balls in a bag and draws one without looking, find each probability.**

$P(m) = \frac{1}{8}$ or 0.125 $\quad$ $P(\text{vowel}) = \frac{3}{8}$ or 0.375

$P(\text{not } a) = \frac{6}{8}$ or 0.75 $\quad$ $P(w) = \frac{0}{8}$ or 0

$P(a \text{ or } n) = \frac{4}{8}$ or 0.5 $\quad$ $P(m, a, r, y, n, \text{ or } e) = \frac{8}{8}$ or 1

A box contains 6 black crayons, 4 blue crayons, 5 red crayons, 3 yellow crayons, and 2 white crayons. If one crayon is chosen without looking, find the probability of each event.

1. $P(\text{black})$ **2.** $P(\text{blue})$ **3.** $P(\text{not white})$ **4.** $P(\text{pink})$

5. $P(\text{black or blue})$ **6.** $P(\text{red, yellow, or white})$

The numbers from 1 to 25 are written on slips of paper and one is drawn without looking. Find the probability of each event.

7. $P(\text{an odd number})$ **8.** $P(\text{a three digit number})$ **9.** $P(\text{not } 4)$

10. $P(\text{a positive number})$ **11.** $P(\text{a prime number})$ **12.** $P(> 19)$

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6-7 Study Guide

Least Common Multiple

A multiple of a number is the product of that number and any whole number. The least nonzero multiple of two or more numbers is the least common multiple (LCM) of the numbers.

Example 1 **Find the least common multiple of 12 and 15.**

multiples of 12: 0, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, . . .
multiples of 15: 0, 15, 30, 45, 60, 75, 90, 105, 120, 135, . . .
60 and 120 are common multiples. The LCM is 60.

Prime factorization can also be used to find the LCM.

Example 2 **Find the least common multiple of 15, 28, and 30.**

$15 = 3 \cdot 5$
$28 = 2 \cdot 2 \cdot 7$
$30 = 2 \cdot 3 \cdot 5$

Find the prime factors of each number.

2, 3, 5 — *Find the common factors.*

$2 \cdot 3 \cdot 5 \cdot 2 \cdot 7 = 420$ — *Multiply the common factors and any other factors.*

The LCM of 15, 28, and 30 is 420.

Find the LCM of each set of numbers.

1. 9, 15 **2.** 16, 12 **3.** 42, 12 **4.** 6, 10

5. 21, 15 **6.** 15, 20 **7.** 9, 15, 18 **8.** 4, 10, 8

9. 12, 15, 24 **10.** 30, 21, 7 **11.** 13, 52, 24 **12.** 8, 14, 28

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6-8 Study Guide

Comparing and Ordering Rational Numbers

One way to compare rational numbers is to express them as fractions with like denominators.

Example 1 Replace ○ with <, >, or = to make $\frac{6}{7}$ ○ $\frac{7}{9}$ a true sentence.

Method 1 Use fractions.

The least common denominator (LCD) of 7 and 9 is 63.

$$\frac{6}{7} \overset{\times 9}{=} \frac{54}{63} \qquad \frac{7}{9} \overset{\times 7}{=} \frac{49}{63}$$

$\frac{54}{63} > \frac{49}{63}$; so $\frac{6}{7} > \frac{7}{9}$.

Method 2 Use decimals.

6 $\boxed{\div}$ 7 $\boxed{=}$ 0.857142857

7 $\boxed{\div}$ 9 $\boxed{=}$ 0.777777778

0.857 > 0.778, so $\frac{6}{7} > \frac{7}{9}$.

Another way to compare rational numbers is to express them as decimals. Then compare the decimals.

Example 2 **Order $\frac{4}{9}$, $\frac{2}{5}$, $0.\overline{47}$, and 0.45 from least to greatest.**

$\frac{4}{9} = 4 \div 9$ or $0.\overline{4}$ $\qquad$ $\frac{2}{5} = 2 \div 5$ or 0.4

The rational numbers in order from least to greatest are:

$\frac{2}{5}$, $\frac{4}{9}$, 0.45, $0.\overline{47}$.

Replace each ○ with <, >, or = to make a true sentence.

1. 3.7 ○ $3.\overline{7}$ $\qquad$ **2.** $\frac{5}{8}$ ○ $\frac{7}{11}$ $\qquad$ **3.** −6.8 ○ $-6\frac{4}{5}$

4. 0.555 ○ $\frac{5}{9}$ $\qquad$ **5.** $6\frac{7}{12}$ ○ 6.6 $\qquad$ **6.** $\frac{3}{10}$ ○ $\frac{5}{16}$

Order each set of rational numbers from least to greatest.

7. 0.67, 0.6, 0.7, 0.06 $\qquad$ **8.** $\frac{3}{7}, \frac{2}{5}, \frac{5}{8}, \frac{1}{2}$ $\qquad$ **9.** $-\frac{1}{3}, \frac{7}{8}, -\frac{9}{10}, \frac{13}{14}$

10. $\frac{15}{13}$, 1.2, $\frac{19}{18}$ $\qquad$ **11.** $-\frac{1}{6}, \frac{1}{6}, -\frac{2}{15}, \frac{1}{9}$ $\qquad$ **12.** 1.77, $1\frac{5}{6}$, $1\frac{1}{2}$, 1.46

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6-9 Study Guide

Scientific Notation

A number in **scientific notation** is written as the product of a number between 1 and 10 and a power of ten.

Examples

1 Express 8.65×10^7 in standard form.

$$\begin{aligned} 8.65 \times 10^7 &= 8.65 \times 10{,}000{,}000 \\ &= 8.6500000. && \textit{Move the decimal point 7 places to the right.} \\ &= 86{,}500{,}000 \end{aligned}$$

2 Express 9.1×10^{-4} in standard form.

$$\begin{aligned} 9.1 \times 10^{-4} &= 9.1 \times \frac{1}{10^4} \\ &= 9.1 \times \frac{1}{10{,}000} \\ &= 9.1 \times 0.0001 \\ &= 0.0009.1 && \textit{Move the decimal point 4 places to the left.} \\ &= 0.00091 \end{aligned}$$

3 Express 1,088,000 in scientific notation.

1.088000. — *Move the decimal point to the right of the first nonzero digit. Move the decimal point 6 places to the left.*

$1{,}088{,}000 = 1.088 \times 10^6$

4 Express 0.0000762 in scientific notation.

0.00007.62 — *Move the decimal point to the right of the first nonzero digit. Move the decimal point 5 places to the right.*

$0.0000762 = 7.62 \times 10^{-5}$

Express each number in standard form.

1. 7.02×10^4

2. 1.1×10^{-3}

3. 6.4×10^7

4. 5.9×10^8

5. 9.12×10^{-2}

6. 8.8×10^{-5}

Express each number in scientific notation.

7. 0.0003

8. 4,600,000

9. 0.00001653

10. 518,900,000

11. 720

12. 0.114