



Study Guide

Adding and Subtracting Like Fractions

To add fractions with like denominators, add the numerators.

Examples 1 Solve $m = \frac{7}{8} + \left(-\frac{5}{8}\right)$.

$$m = \frac{2}{8} \quad \text{Add the numerators.}$$

$$m = \frac{1}{4} \quad \text{Simplify.}$$

2 Solve $n = \frac{5}{6} + \frac{5}{6}$.

$$n = \frac{10}{6} \quad \text{Add the numerators.}$$

$$n = 1\frac{4}{6} \quad \text{Rename the improper fraction as a mixed number.}$$

$$n = 1\frac{2}{3} \quad \text{Simplify.}$$

To subtract fractions with like denominators, subtract the numerators.

Examples 3 Solve $a = \frac{8}{9} - \frac{5}{9}$.

$$a = \frac{3}{9} \quad \text{Subtract the numerators.}$$

$$a = \frac{1}{3} \quad \text{Simplify.}$$

4 Solve $b = -\frac{5}{8} - \left(-\frac{3}{8}\right)$.

$$b = -\frac{2}{8} \quad \text{Subtract the numerators.}$$

$$b = -\frac{1}{4} \quad \text{Simplify.}$$

Solve each equation. Write the solution in simplest form.

1. $\frac{4}{7} + \frac{2}{7} = f$

2. $\frac{3}{4} - \frac{1}{4} = p$

3. $a = \frac{5}{8} + \frac{5}{8}$

4. $z = -\frac{2}{3} + \frac{1}{3}$

5. $-\frac{5}{12} - \frac{1}{12} = t$

6. $v = -\frac{6}{11} + \left(-\frac{5}{11}\right)$

7. $-\frac{1}{8} - \left(-\frac{3}{8}\right) = k$

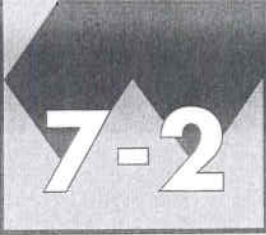
8. $\frac{14}{15} + \frac{6}{15} = x$

9. $c = -\frac{1}{2} - \frac{1}{2}$

10. $j = -\frac{12}{13} + \frac{5}{13}$

11. $-\frac{13}{16} + \left(-\frac{11}{16}\right) = k$

12. $r = -\frac{2}{3} - \left(-\frac{2}{3}\right)$



Study Guide

Adding and Subtracting Unlike Fractions

To add or subtract fractions or mixed numbers with unlike denominators, rename the fractions with a common denominator. Then add or subtract.

Examples 1 Solve $a = -\frac{5}{8} + \left(-\frac{3}{4}\right)$. The least common denominator of 8 and 4 is 8.

$$a = -\frac{5}{8} + \left(-\frac{6}{8}\right) \quad \text{Rename } -\frac{3}{4} \text{ as } -\frac{6}{8}.$$

$$a = -\frac{11}{8} \quad \text{Add.}$$

$$a = -1\frac{3}{8} \quad \text{Rename the improper fraction as a mixed number.}$$

2 Solve $c = -2\frac{3}{5} - 1\frac{1}{2}$. The least common denominator of 5 and 12 is 10.

$$c = -2\frac{6}{10} - 1\frac{5}{10} \quad \text{Rename } \frac{3}{5} \text{ as } \frac{6}{10}. \text{ Rename } \frac{1}{2} \text{ as } \frac{5}{10}.$$

$$c = -3\frac{11}{10} \quad \text{Subtract.}$$

$$c = -4\frac{1}{10} \quad \text{Rename } \frac{11}{10} \text{ as } 1\frac{1}{10}.$$

3 Solve $r = 5\frac{1}{4} - 2\frac{2}{3}$. The least common denominator of 4 and 3 is 12.

$$r = 5\frac{3}{12} - 2\frac{8}{12} \quad \text{Rename } \frac{1}{4} \text{ as } \frac{3}{12}. \text{ Rename } \frac{2}{3} \text{ as } \frac{8}{12}.$$

$$r = 4\frac{15}{12} - 2\frac{8}{12} \quad \text{Rename } 5\frac{3}{12} \text{ as } 4\frac{15}{12}.$$

$$r = 2\frac{7}{12} \quad \text{Subtract.}$$

Solve each equation. Write the solution in simplest form.

1. $n = \frac{3}{4} + \frac{1}{3}$

2. $\frac{7}{8} - \frac{2}{3} = k$

3. $-\frac{11}{12} - \frac{1}{2} = y$

4. $1\frac{1}{2} + \left(-1\frac{1}{5}\right) = v$

5. $x = -3\frac{2}{3} + \left(-1\frac{1}{6}\right)$

6. $m = 10\frac{11}{12} + 9\frac{3}{8}$

7. $p = 7\frac{1}{3} - \left(-2\frac{5}{9}\right)$

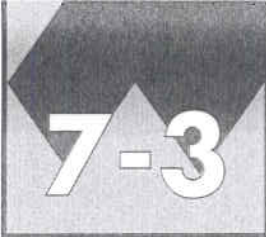
8. $-\frac{15}{16} - \frac{3}{8} = f$

9. $3\frac{4}{5} - \left(-5\frac{1}{2}\right) = c$

10. $2\frac{3}{4} + \left(-6\frac{3}{8}\right) = a$

11. $-9\frac{5}{6} - \left(-3\frac{2}{3}\right) = q$

12. $m = \frac{5}{9} - \frac{1}{3}$



Study Guide

Multiplying Fractions

To multiply fractions, multiply the numerators and multiply the denominators. Use the rules for multiplying integers when you multiply negative fractions.

Example 1 Solve $k = -\frac{4}{7} \times \frac{5}{9}$.

$$k = -\frac{4 \times 5}{7 \times 9}$$

$$k = -\frac{20}{63}$$

*Multiply the numerators.
Multiply the denominators.*

The product of two rational numbers with different signs is negative.

Example 2 Solve $n = 3\frac{1}{3} \times 2\frac{1}{5}$.

$$n = \frac{10}{3} \times \frac{11}{5}$$

$$n = \frac{2 \times 11}{3 \times 1} = \frac{22}{3}$$

$$n = 7\frac{1}{3}$$

*Rename $3\frac{1}{3}$ as $\frac{10}{3}$. Rename $2\frac{1}{5}$ as $\frac{11}{5}$.
The GCF of 10 and 5 is 5. Divide 10 and 5 by 5.*

*Multiply the numerators.
Multiply the denominators.*

Simplify.

Solve each equation. Write the solution in simplest form.

1. $k = \frac{2}{3} \times \frac{3}{5}$

2. $-\frac{1}{2} \times \frac{7}{9} = m$

3. $-\frac{4}{7} \times \left(-\frac{7}{8}\right) = n$

4. $1\frac{1}{2} \times 1\frac{2}{3} = v$

5. $x = -2\frac{1}{4} \times \frac{2}{9}$

6. $r = -8 \times \left(-\frac{3}{4}\right)$

7. $p = \frac{3}{8} \times \left(-2\frac{2}{3}\right)$

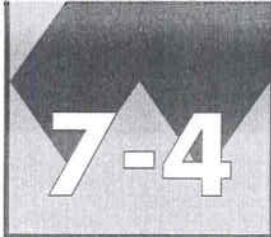
8. $6\frac{1}{2} \times \frac{4}{5} = w$

9. $9 \times \left(-2\frac{2}{3}\right) = h$

10. $4\left(5\frac{3}{4}\right) = f$

11. $c = \left(\frac{1}{2}\right)^2$

12. $t = \frac{4}{7} \times \left(-\frac{2}{3}\right)$



Study Guide

Properties of Rational Numbers

Use the basic properties for addition and multiplication of rational numbers to help you evaluate expressions.

Commutative Property

For any numbers a and b :

$$a + b = b + a$$

$$a \times b = b \times a$$

Associative Property

For any numbers a , b , and c :

$$(a + b) + c = a + (b + c)$$

$$(a \times b) \times c = a \times (b \times c)$$

Identity Property

For any number a :

$$a + 0 = a$$

$$a \times 1 = a$$

Examples

$$\frac{1}{4} + \frac{2}{3} = \frac{2}{3} + \frac{1}{4}$$

$$\frac{4}{5} \times \frac{1}{2} = \frac{1}{2} \times \frac{4}{5}$$

Examples

$$\left(-\frac{1}{3} + \frac{2}{5}\right) + \frac{1}{2} = -\frac{1}{3} + \left(\frac{2}{5} + \frac{1}{2}\right)$$

$$\left(\frac{5}{7} \times -\frac{3}{4}\right) \times \frac{1}{3} = \frac{5}{7} \times \left(-\frac{3}{4} \times \frac{1}{3}\right)$$

Examples

$$\frac{7}{8} + 0 = \frac{7}{8}$$

$$-\frac{11}{12} \times 1 = -\frac{11}{12}$$

Distributive Property

For any numbers a , b , and c :

$$a \times (b + c) = (a \times b) + (a \times c)$$

Example

$$4 \times \left(7 + \frac{5}{8}\right) = \left(4 \times 7\right) + \left(4 \times \frac{5}{8}\right)$$

Inverse Property of Multiplication

For any numbers a and b , with $a, b \neq 0$:

$$\frac{a}{b} \times \frac{b}{a} = 1$$

Example

$\frac{2}{5} \times \frac{5}{2} = 1$ $\frac{2}{5}$ and $\frac{5}{2}$ are reciprocals or multiplicative inverses of each other.

Name the multiplicative inverse of each rational number.

1. 8

2. -7

3. $\frac{3}{4}$

4. $-\frac{9}{15}$

5. 0.5

6. $\frac{r}{t}$

7. $-p$

8. $2\frac{1}{4}$

9. $-\frac{m}{n}$

10. k

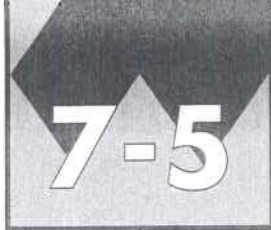
Evaluate each expression if $a = \frac{1}{4}$, $b = \frac{3}{8}$, $x = -1\frac{1}{2}$, and $y = 2\frac{2}{3}$.

11. ab

12. $2x$

13. by

14. a^2



Study Guide

Integration: Patterns and Functions Sequences

A list of numbers in a specific order is a **sequence**. Each number is called a **term** of the sequence. When the difference between any two consecutive terms is the same, the sequence is an **arithmetic sequence**.

Examples $0 \quad 2\frac{1}{4} \quad 4\frac{1}{2} \quad 6\frac{3}{4} \quad 9, \dots$ The difference between consecutive terms is $2\frac{1}{4}$.

$+ 2\frac{1}{4} \quad + 2\frac{1}{4} \quad + 2\frac{1}{4} \quad + 2\frac{1}{4}$

When consecutive terms are formed by multiplying by the same factor, the sequence is **geometric**.

Example $8, \quad 4, \quad 2, \quad 1, \quad \frac{1}{2}, \dots$ Multiply by $\frac{1}{2}$ to find consecutive terms.

$\times \frac{1}{2} \quad \times \frac{1}{2} \quad \times \frac{1}{2} \quad \times \frac{1}{2}$

Some sequences are neither arithmetic or geometric.

Example $100, \quad 80, \quad 62, \quad 46, \quad 32, \dots$ There is no common difference or common multiplier.

$-20 \quad -18 \quad -16 \quad -14$

Identify each sequence as arithmetic, geometric, or neither. Then write the next three terms.

1. $47, 42, 37, 32, \dots$

2. $9, 18, 36, 72, \dots$

3. $\frac{1}{9}, \frac{1}{3}, 1, 3, 9, \dots$

4. $15, 17, 20, 24, 29, \dots$

5. $8, 7\frac{3}{5}, 7\frac{1}{5}, 6\frac{4}{5}, \dots$

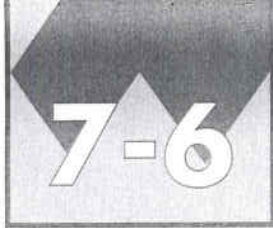
6. $8, 2, \frac{1}{2}, \frac{1}{8}, \dots$

7. $-9, -5, -1, 3, 7, \dots$

8. $\frac{4}{7}, \frac{2}{7}, 0, -\frac{2}{7}, \dots$

9. $1, 1, 2, 3, 5, 8, \dots$

10. $\frac{1}{10}, \frac{1}{2}, 2\frac{1}{2}, 12\frac{1}{2}, \dots$



Study Guide

Integration: Geometry Area of Triangles and Trapezoids

The area of a triangle is equal to one-half its base times its height:

$$A = \frac{1}{2}bh.$$

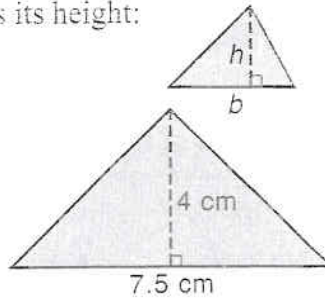
Example 1 Find the area of the triangle.

$$A = \frac{1}{2}bh.$$

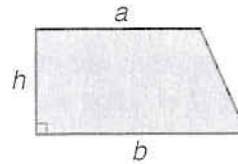
$$A = \frac{1}{2}(7.5)(4) \quad b = 7.5, h = 4$$

$$A = 2(7.5)$$

$$A = 15 \quad \text{The area is 15 square centimeters.}$$



The area of a trapezoid is equal to one-half its height times the sum of its bases: $A = \frac{1}{2}h(a + b)$.



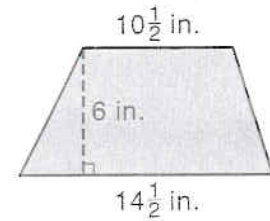
Example 2 Find the area of the trapezoid.

$$A = \frac{1}{2}h(a + b)$$

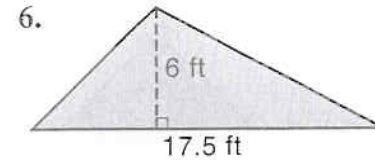
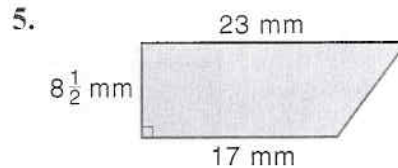
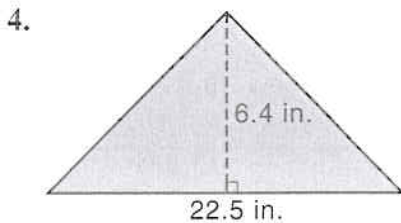
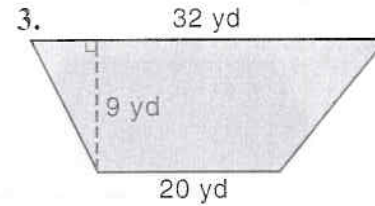
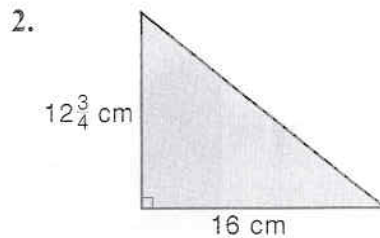
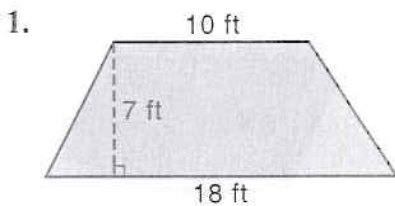
$$A = \frac{1}{2}(6)\left(10\frac{1}{2} + 14\frac{1}{2}\right) \quad h = 6, a = 10\frac{1}{2}, b = 14\frac{1}{2}$$

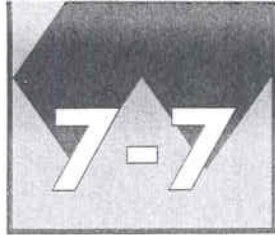
$$A = 3(25)$$

$$A = 75 \quad \text{The area is 75 square inches.}$$



State the measures of the base(s) and the height of each triangle or trapezoid. Then find the area of each figure.





Study Guide

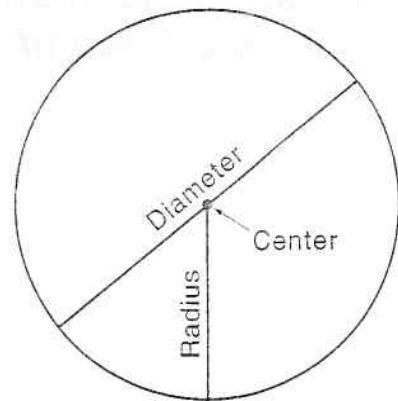
Integration: Geometry Circles and Circumference

A circle is a set of points in a plane that are the same distance from a given point called the center.

The diameter (d) is the distance across the circle through its center.

The radius (r) is the distance from the center to any point on the circle.

The circumference (C) is the distance around the circle.



Examples 1 Find the circumference of a circle with a diameter of 9.5 inches.

$$C = \pi d$$

$$C = 3.14 \times 9.5 \quad \text{Use } 3.14 \text{ for } \pi.$$

$$C \approx 29.83$$

The circumference of the circle is about 29.83 inches.

2 Find the circumference of a circle with a radius of 14 inches.

$$C = 2\pi r$$

$$C = 2 \times \frac{22}{7} \times 14 \quad \text{Use } \frac{22}{7} \text{ for } \pi.$$

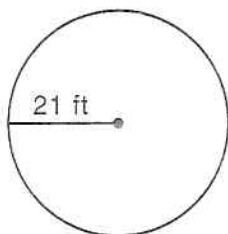
$$C \approx 88$$

The circumference of the circle is approximately 88 inches.

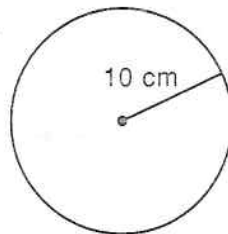
Find the circumference of each circle to the nearest tenth.

Use $\frac{22}{7}$ or 3.14 for π .

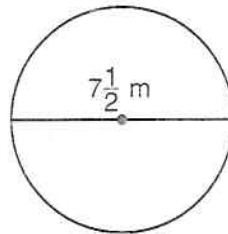
1.



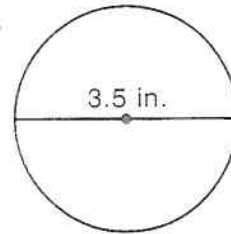
2.



3.



4.



5. The radius is 42 miles.

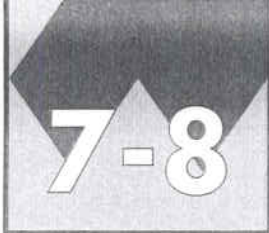
6. The diameter is 68 meters.

7. The diameter is 700 feet.

8. The radius is $7\frac{3}{4}$ inches.

9. The radius is 91 centimeters.

10. The diameter is $2\frac{1}{3}$ kilometers.



Study Guide

Dividing Fractions

To divide by a fraction, multiply by its multiplicative inverse. Use the rules for dividing integers when you divide negative fractions.

Examples 1 Solve $m = 24 \div \frac{3}{4}$.

$$m = \frac{24}{1} \div \frac{3}{4}$$

Rename 24 as $\frac{24}{1}$.

$$m = \frac{24}{1} \times \frac{4}{3}$$

Multiply by $\frac{4}{3}$, the multiplicative inverse of $\frac{3}{4}$.

$$m = 32$$

2 Solve $p = -2\frac{1}{3} \div \frac{5}{6}$.

$$p = -\frac{7}{3} \div \frac{5}{6}$$

Rename $-2\frac{1}{3}$ as $-\frac{7}{3}$.

$$p = -\frac{7}{3} \times \frac{6}{5}$$

Multiply by $\frac{6}{5}$, the multiplicative inverse of $\frac{5}{6}$.

$$p = -\frac{14}{5}$$

The product of a negative number and a positive number is negative.

$$p = -2\frac{4}{5}$$

Rename $-\frac{14}{5}$ as $-2\frac{4}{5}$.

3 Solve $r = -\frac{7}{8} \div \left(-4\frac{1}{2}\right)$.

$$r = -\frac{7}{8} \div \left(-\frac{9}{2}\right)$$

Rename $-4\frac{1}{2}$ as $-\frac{9}{2}$.

$$r = -\frac{7}{8} \times \left(-\frac{2}{9}\right)$$

Multiply by $-\frac{2}{9}$, the multiplicative inverse of $-\frac{9}{2}$.

$$r = \frac{7}{36}$$

The product of two negative numbers is positive.

Solve each equation. Write the solution in simplest form.

1. $y = \frac{4}{5} \div \frac{1}{10}$

2. $15 \div \frac{5}{8} = k$

3. $r = -25 \div 1\frac{3}{7}$

4. $5\frac{1}{3} \div \left(-\frac{3}{8}\right) = t$

5. $f = -\frac{15}{16} \div \left(-\frac{3}{4}\right)$

6. $7\frac{1}{2} \div \left(-2\frac{1}{2}\right) = y$

Evaluate each expression.

7. $a \div b$ if $a = -10\frac{5}{6}$ and $b = 4\frac{1}{3}$

8. $r \div (s + t)$ if $r = 10$, $s = -4\frac{3}{8}$, $t = -1\frac{5}{8}$

9. $a^2 \div y$ if $a = -9$ and $y = -\frac{1}{3}$



Study Guide

Solving Equations

Solve equations containing rational numbers the same way you solve integer equations.

Examples 1 Solve $-\frac{2}{3}m = \frac{10}{21}$.

$$-\frac{3}{2} \times -\frac{2}{3}m = -\frac{3}{2} \times \frac{10}{21}$$
$$m = -\frac{5}{7}$$

Multiply each side by $-\frac{3}{2}$, the multiplicative inverse of $-\frac{2}{3}$.

Check: $-\frac{2}{3} \times \left(-\frac{5}{7}\right) \stackrel{?}{=} \frac{10}{21}$

$$\frac{10}{21} = \frac{10}{21} \quad \checkmark$$

2 Solve $-\frac{3}{8} = \frac{t}{5}$.

$$5 \times \left(-\frac{3}{8}\right) = 5 \times \frac{t}{5} \quad \text{Multiply each side by 5.}$$

$$-\frac{15}{8} = t$$

$$-1\frac{7}{8} = t$$

Check: $-\frac{3}{8} \stackrel{?}{=} -\frac{15}{8} \div 5$

$$-\frac{3}{8} \stackrel{?}{=} -\frac{15}{8} \times \frac{1}{5}$$
$$-\frac{3}{8} = -\frac{3}{8} \quad \checkmark$$

Solve each equation. Check your solution.

1. $3m = -84$

2. $\frac{y}{5} = -9$

3. $-\frac{1}{2}p = 4.6$

4. $c \div 0.2 = 12$

5. $\frac{k}{1.2} = -5.5$

6. $y - \frac{3}{5} = -6$

7. $\frac{v}{8} = -3.2$

8. $\frac{t}{1.5} = -8$

9. $4.9 = \frac{x}{0.3}$

10. $3\frac{1}{2}n = 7\frac{7}{8}$

11. $-\frac{2}{3} = 8k$

12. $\frac{7}{9}f = 9\frac{1}{3}$

Study Guide

Solving Inequalities

You can solve inequalities that involve rational numbers by using the same steps you used to solve inequalities with integers.

Example Solve $5 + \frac{1}{3}y < 2\frac{1}{2}$.

$$5 + \frac{1}{3}y < 2\frac{1}{2}$$

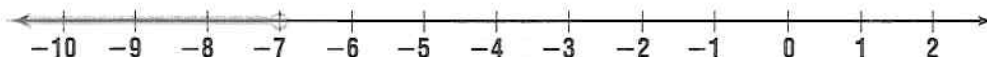
$$5 + \frac{1}{3}y - 5 < 2\frac{1}{2} - 5$$

$$\frac{1}{3}y < -2\frac{1}{2}$$

$$3\left(\frac{1}{3}y\right) < 3\left(-2\frac{1}{2}\right)$$

$$y < -7\frac{1}{2}$$

The numbers less than $-7\frac{1}{2}$ make up the solution set. The number line below shows the solution.

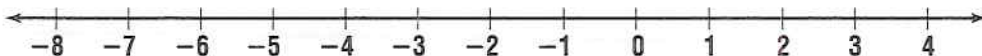


Solve each inequality. Graph the solution on a number line.

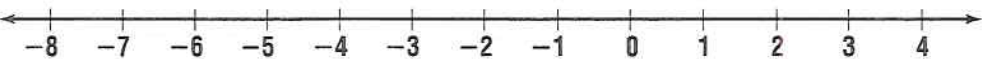
1. $a + 2\frac{1}{3} < -\frac{1}{6}$



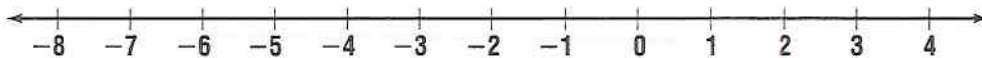
2. $1\frac{1}{8} - b \geq \frac{7}{16}$



3. $-0.4c \leq 0.9$



4. $\frac{f - 6.25}{2.5} < -3.5$



5. $\frac{3}{2}d - 4 \geq \frac{7}{8}$



6. $\frac{-\frac{4}{3} - y}{\frac{5}{9}} < 2\frac{1}{5}$

