

## Study Guide

### Using Proportions

You can use proportions to solve problems.

**Example** Of the 360 members of the kennel club who were surveyed, 190 said they would try a new dog food. If the kennel club has 2,100 members, how many could be expected to try the new dog food?

Let  $t$  represent the number of members who might try the new dog food. Write a proportion.

$$\frac{190}{360} = \frac{t}{2,100} \quad \begin{array}{l} \leftarrow \text{will try new dog food} \\ \leftarrow \text{kennel club members} \end{array}$$

$$190 \cdot 2,100 = 360t \quad \text{Find the cross products.}$$

$$399,000 = 360t$$

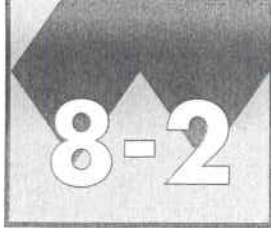
$$\frac{399,000}{360} = \frac{360t}{360}$$

$$1,108 \approx t$$

About 1,108 members of the kennel club might be expected to try the new dog food.

### Solve.

1. A shop produces 47 surfboards in 6 days. How long will it take them to make 423 surfboards?
2. Six bottles of mineral water cost \$2.29. How much will 84 bottles of mineral water cost?
3. A recipe for muffins calls for  $1\frac{1}{2}$  cups of bran cereal. The recipe makes 12 muffins. How much bran cereal is needed to make 96 muffins?
4. Five out of every 8 people surveyed in Carson City approved of a new golf course. If there are 26,000 people in Carson City, about how many can be expected to approve of the golf course?
5. Ryan's heart beats 216 times in 3 minutes. How many times will it beat in an hour?
6. A sample of fish from a river showed that  $\frac{2}{5}$  of the fish were less than one year old. If there are an estimated 8,000 fish in the river, about how many of them are less than one year old?
7. Maria drove her car 250 miles on 10 gallons of gasoline. About how many gallons of gasoline will she need for a 1,250-mile trip?



# Study Guide

## The Percent Proportion

You can use a proportion to find the percent for a fraction.

**Example 1** Express  $\frac{13}{8}$  as a percent.

$$\begin{aligned}\frac{13}{8} &= \frac{x}{100} \\ 1,300 &= 8x \\ 162.5 &= x \\ \frac{13}{8} &= 162.5\%\end{aligned}$$

You can use the percent proportion to solve problems.

$$\frac{P}{B} = \frac{r}{100} \text{ or } \frac{\text{Percentage}}{\text{Base}} = \frac{\text{rate}}{100}$$

**Examples 2** 37.2 is what percent of 186?

$$\begin{aligned}\frac{P}{B} &= \frac{r}{100} \\ \frac{37.2}{186} &= \frac{r}{100} \\ 3,720 &= 186r \\ 20 &= r \\ 37.2 &\text{ is } 20\% \text{ of } 186.\end{aligned}$$

**3** What number is 15% of 280?

$$\begin{aligned}\frac{P}{B} &= \frac{r}{100} \\ \frac{P}{280} &= \frac{15}{100} \\ 100P &= 4,200 \\ P &= 42 \\ 42 &\text{ is } 15\% \text{ of } 280.\end{aligned}$$

**Express each fraction as a percent.**

1.  $\frac{23}{100}$

2.  $\frac{3}{4}$

3.  $\frac{11}{20}$

4.  $\frac{13}{25}$

5.  $\frac{29}{50}$

6.  $\frac{7}{8}$

7.  $\frac{7}{16}$

8.  $\frac{7}{25}$

9.  $\frac{2}{5}$

10.  $\frac{13}{40}$

**Write a percent proportion to solve each problem. Then solve. Round to the nearest tenth.**

11. What is 40% of 160?

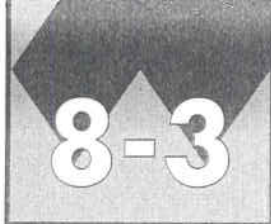
12. 75 is what percent of 375?

13. 45 is 25% of what number?

14. 14.5 is 5% of what number?

15. Find 12% of 260.

16. 63 is what percent of 420?



## Study Guide

### Integration: Algebra The Percent Equation

To solve percent problems, you can use the equation  $P = RB$ , where  $P$  = percentage,  $R$  = rate, and  $B$  = base.

**Examples** 1 What number is 16% of 245? Estimate:  $16\% \approx \frac{1}{5}$ ;  $\frac{1}{5} \times 250 = 50$ .

$$P = RB$$

$$P = 0.16(245)$$

$$P = 39.2$$

Replace  $R$  with 16% or 0.16 and  $B$  with 245.

39.2 is 16% of 245.

Compare to the estimate.

2 45 is 15% of what number? Estimate:  $15\% \approx \frac{1}{5}$ ; 45 is  $\frac{1}{5}$  of 225.

$$P = RB$$

$$45 = 0.15B$$

$$45 \div 0.15 = 0.15B \div 0.15$$

$$300 = B$$

Replace  $P$  with 45 and  $R$  with 15% or 0.15.

Divide each side by 0.15.

45 is 15% of 300.

Compare to the estimate.

3 What percent of 750 is 60?

$$P = RB$$

$$60 = R(750)$$

$$60 \div 750 = R(750) \div 750$$

$$0.08 = R$$

Replace  $P$  with 60 and  $B$  with 750.

Divide each side by 750.

8% of 750 is 60.

#### Solve.

1. What percent of 80 is 12?

2. What is 18% of 110?

3. Find 2% of 1,600.

4. 75 is 20% of what number?

5. What percent of 42 is 63?

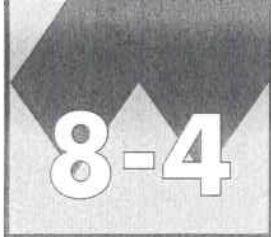
6. What is 5.5% of 90?

7. 64% of what number is 288?

8. Find 88% of \$17.50.

9. What is 115% of 90?

10. \$5 is what percent of \$70?



# Study Guide

## Large and Small Percents

To express a percent greater than 100% or less than 1% as a fraction, write the percent as a fraction with a denominator of 100 and simplify.

**Examples**

<b>1</b>	$0.4\% = \frac{0.4}{100}$	<b>2</b>	$175\% = \frac{175}{100}$	<b>3</b>	$\frac{1}{6}\% = \frac{1}{600}$
	$= \frac{4}{1,000}$		$= 1\frac{75}{100}$		$= \frac{1}{6} \div 100$
	$= \frac{1}{250}$		$= 1\frac{3}{4}$		$= \frac{1}{6} \times \frac{1}{100}$
					$= \frac{1}{600}$

To express a percent greater than 100% or less than 1% as a decimal, write the percent as a fraction with a denominator of 100 and then express the fraction as a decimal.

**Examples**

<b>4</b>	$254\% = \frac{254}{100}$	<b>5</b>	$0.05\% = \frac{0.05}{100}$	<b>6</b>	$\frac{5}{8}\% = \frac{5}{800}$
	$= 2.54$		$= \frac{5}{10,000}$		$= \frac{5}{8} \div 100$
			$= 0.0005$		$= \frac{5}{8} \times \frac{1}{100}$
					$= \frac{5}{800}$
					$= 0.00625$

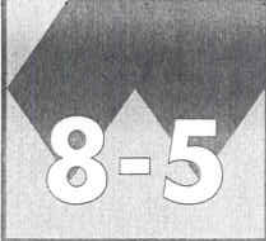
**Express each percent as a fraction or mixed number in simplest form.**

- |         |                    |                     |           |
|---------|--------------------|---------------------|-----------|
| 1. 0.8% | 2. 0.09%           | 3. 853%             | 4. 420.5% |
| 5. 674% | 6. $\frac{1}{2}\%$ | 7. $\frac{3}{50}\%$ | 8. 0.15%  |

**Express each percent as a decimal.**

- |                      |            |            |                     |
|----------------------|------------|------------|---------------------|
| 9. 716%              | 10. 0.07%  | 11. 1,463% | 12. 0.9%            |
| 13. $\frac{3}{10}\%$ | 14. 0.004% | 15. 900%   | 16. $\frac{3}{5}\%$ |





# Study Guide

## Percent of Change

To find the percent of increase or decrease, first find the amount of the increase or decrease. Then find the ratio of that amount to the original amount and express it as a percent.

**Example 1** Two years ago a bicycle shop sold 675 bicycles. This year, 865 bicycles were sold. To the nearest percent, what is the percent of increase?

First, find the amount of change:  $865 - 675 = 190$ .  
Then, find the percent using 675 as the base.

$$\frac{190}{675} = \frac{r}{100}$$

$$190 \cdot 100 = 675 \cdot r$$

$$19,000 = 675r$$

$$28 \approx r \quad \text{The percent of change is 28\%}.$$

A percent of discount is a percent of decrease.

**Example 2** A VCR that usually sells for \$365 is on sale for 20% off. What is the sale price?

### Method 1

First, find the amount of the discount.

$$RB = P$$

$$0.20 \cdot 365 = P$$

$$73 = P$$

Then, subtract the discount from the regular price.

$$365 - 73 = 292$$

The sale price is \$292.

### Method 2

First, find the percent paid.

$$100\% - 20\% = 80\%$$

Then, find the sale price.

$$RB = P$$

$$0.80 \cdot 365 = P$$

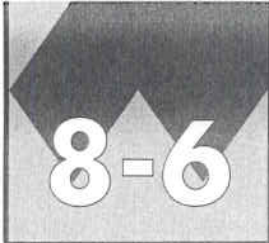
$$292 = P$$

**Find each percent of change. Round to the nearest percent.**

- |                               |                             |                              |                                |
|-------------------------------|-----------------------------|------------------------------|--------------------------------|
| 1. original: \$10<br>new: \$8 | 2. original: 85<br>new: 112 | 3. original: 120<br>new: 200 | 4. original: \$75<br>new: \$30 |
|-------------------------------|-----------------------------|------------------------------|--------------------------------|

**Find the sale price of each item to the nearest cent.**

- |                              |                             |
|------------------------------|-----------------------------|
| 5. \$465 golf clubs, 20% off | 6. \$129 telephone, 30% off |
| 7. \$17.99 video, 40% off    | 8. \$395 guitar, 10% off    |



# Study Guide

## Simple Interest

To find simple interest, use the formula  $I = prt$ . Interest ( $I$ ) is the charge for the use of money. Principal ( $p$ ) is the amount of money. Rate ( $r$ ) is a percent of annual interest. Time ( $t$ ) is the time in years that the money is used.

- Examples** 1 Find the interest earned on \$490 deposited in a savings account at  $8\frac{1}{2}\%$  per year for 6 months.

$$I = prt$$

$$I = 490(0.085)\left(\frac{6}{12}\right)$$

$$I = 20.825 \text{ or } \$20.83$$

$$p = \$490, r = 8\frac{1}{2}\%, \text{ or } 0.085,$$

$$t = 6 \text{ out of } 12 \text{ months or } \frac{6}{12}$$

Find the total amount of money in the savings account.

$$\$490.00 + \$20.83 = \$510.83 \quad \text{Add the interest to the original amount.}$$

- 2 Find the annual rate of simple interest if \$3,600 was borrowed and \$4,248 was repaid 18 months later.

The amount of interest was  $\$4,248 - \$3,600$  or \$648.

$$I = prt$$

$$648 = 3,600(r)\left(\frac{18}{12}\right)$$

$$648 = 5,400r$$

$$0.12 = r$$

$$I = \$648, p = \$3,600, t = \frac{18}{12}$$

The interest rate was 0.12 or 12%.

**Find the simple interest to the nearest cent.**

- \$500 at 7% for 4 years
- \$2,800 at 6.5% for 18 months
- \$1,050 at 18% for 3 months
- \$725 at 5.5% for 2 years

**Find the total amount in each account to the nearest cent.**

- \$1,250 at 5% for 1 year
- \$600 at 14.5% for 9 months
- \$276 at 8% for 3 years
- \$4,500 at 9% for 6 months

**Find the annual rate of simple interest.**

- principal: \$1,700; total amount after 3 years: \$2,210
- principal: \$6,300; total amount after 6 months: \$6,772.50

## Study Guide

### Integration: Geometry Similar Polygons

A **polygon** is a closed figure in a plane and is formed by three or more line segments that meet only at their endpoints.

Two polygons are **similar** if their corresponding angles are congruent and their corresponding sides are in proportion.

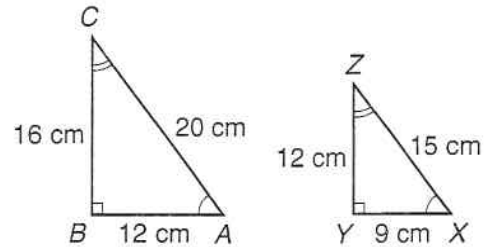
**Example 1**

$$\begin{aligned} \angle A &\cong \angle X & \frac{XY}{AB} &= \frac{9}{12} = \frac{3}{4} \\ \angle B &\cong \angle Y & \frac{YZ}{BC} &= \frac{12}{16} = \frac{3}{4} \\ \angle C &\cong \angle Z & \frac{XZ}{AC} &= \frac{15}{20} = \frac{3}{4} \end{aligned}$$

The corresponding angles are equal.

The corresponding sides are in proportion.

$\triangle ABC \sim \triangle XYZ$  The symbol  $\sim$  means "is similar to."



You can use proportions to solve problems involving similar figures.

**Example 2** The two quadrilaterals are similar.  
Find the length of side  $\overline{VR}$ .

$\overline{PU}$  corresponds to  $\overline{VT}$ .  
 $\overline{PQ}$  corresponds to  $\overline{VR}$ .

Write a proportion.

Substitute values.

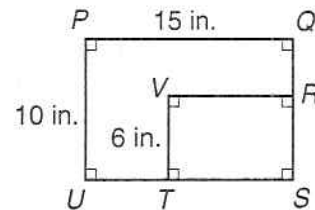
Find cross products.

$$\frac{PU}{VT} = \frac{PQ}{VR}$$

$$\frac{10}{6} = \frac{15}{x}$$

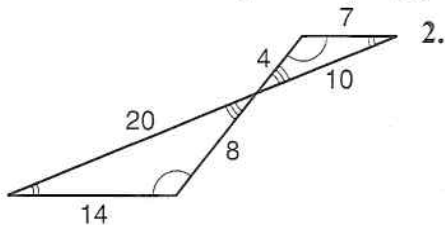
$$\begin{aligned} 10x &= 90 \\ x &= 9 \end{aligned}$$

The length of  $\overline{VR}$  is 9 inches.

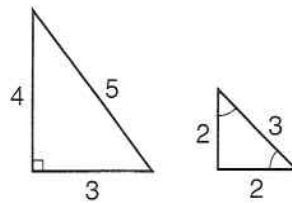


Tell whether each pair of polygons is similar.

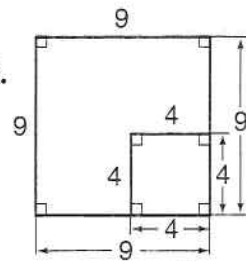
1.



2.



3.

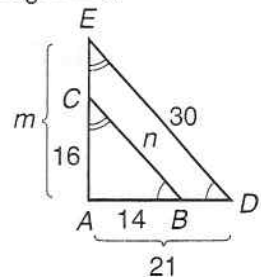


$\triangle ABC \sim \triangle ADE$ . Use this information to answer Exercises 4-6.

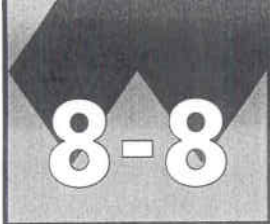
4. List all pairs of corresponding angles.

5. Write a proportion and solve for  $m$ .

6. Write a proportion and solve for  $n$ .







# Study Guide

## Indirect Measurement

You can use a proportion to find a measurement indirectly.

**Example** A bridge from Carl's Cave ( $C$ ) to Bluff Lookout ( $B$ ) across a river is 600 feet long. The distance from Arborville ( $A$ ) to Bluff Lookout is 900 feet. The distance from Arborville to Emmerlyville ( $E$ ) is 1,000 feet. How far is it from Bluff Lookout to Deadman's Rock ( $D$ )?

$$\frac{AC}{BC} = \frac{AE}{BD}$$

Write a proportion.

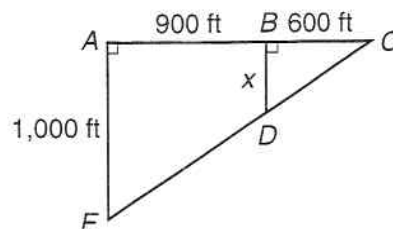
$$\frac{900 + 600}{600} = \frac{1,000}{x}$$

Substitute.

$$1,500x = 600,000$$

Find cross products.

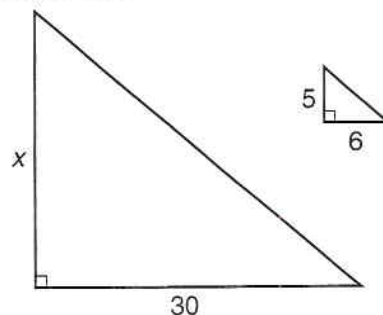
$$x = 400$$



It is 400 feet from Bluff Lookout to Deadman's Rock.

**Write a proportion for each problem and then solve it. Assume the triangles are similar.**

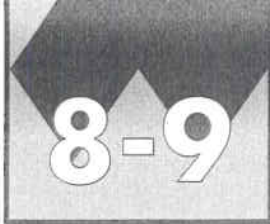
1. A statue casts a shadow 30 feet long. At the same time, a person who is 5 feet tall casts a shadow that is 6 feet long. How tall is the statue?



2. A guide wire is attached to the top of a radio tower. It attaches to the ground 40 feet from its base. Angelica is 5 feet tall. When she stands so her head touches the guide wire, she is 2 feet from the point where it attaches to the ground. How tall is the tower?





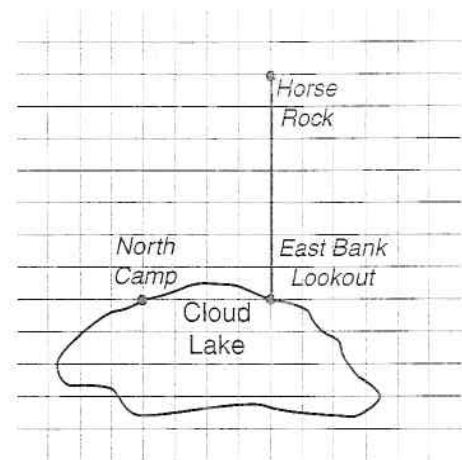


# Study Guide

## Scale Drawings and Models

In a scale drawing, the lengths on the drawing are proportional to the actual lengths. Maps and blueprints are examples of scale drawings.

**Example** On this map, each one-half centimeter square represents 5 kilometers. Find the distance from Horse Rock to East Bank Lookout.



Use the scale to form a proportion.  
Then solve the proportion.

$$\frac{\frac{1}{2} \text{ cm}}{5 \text{ km}} = \frac{3\frac{1}{2} \text{ cm}}{x \text{ km}}$$

$$\frac{1}{2}x = 5\left(3\frac{1}{2}\right)$$

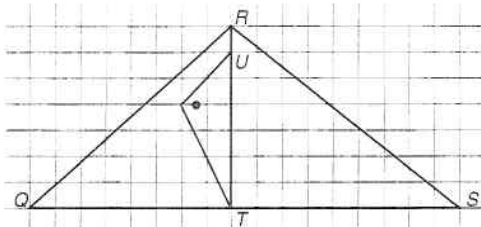
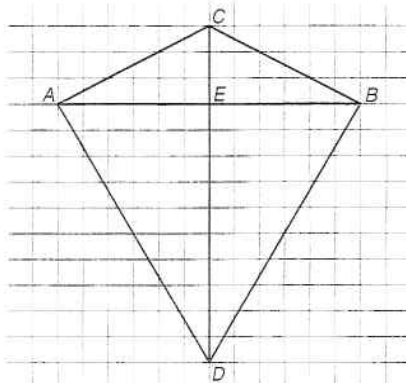
$$\frac{1}{2}x = 17\frac{1}{2}$$

$$x = 35$$

It is 35 kilometers from Horse Rock to East Bank Lookout.

The figures show scale drawings of two types of kites. In the top drawing, the side of each square represents 2 inches. In the bottom drawing, the side of each square represents 3 inches. Find the actual length of each segment.

1.  $\overline{AB}$
2.  $\overline{CD}$
3.  $\overline{CE}$
4.  $\overline{ED}$
5.  $\overline{QT}$
6.  $\overline{QS}$
7.  $\overline{RU}$
8.  $\overline{UT}$



## Study Guide

### Integration: Geometry Dilations

Enlarging or reducing a figure is called a **dilation**. A dilated figure is similar to the original figure. The ratio of the new figure to the original is called the scale factor.

**Example** Graph trapezoid  $ABCD$  with vertices  $A(2, 2)$ ,  $B(8, 4)$ ,  $C(8, 10)$ ,  $D(2, 10)$ . Graph its dilation with a scale factor of 0.5.

To find the vertices of the dilation image, multiply each coordinate in the ordered pairs by 0.5.

$$A(2, 2) \rightarrow (2 \cdot 0.5, 2 \cdot 0.5) \rightarrow A'(1, 1)$$

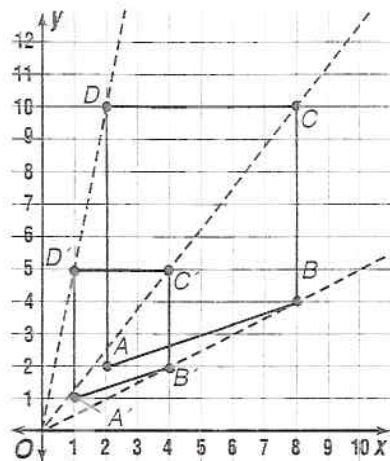
$$B(8, 4) \rightarrow (8 \cdot 0.5, 4 \cdot 0.5) \rightarrow B'(4, 2)$$

$$C(8, 10) \rightarrow (8 \cdot 0.5, 10 \cdot 0.5) \rightarrow C'(4, 5)$$

$$D(2, 10) \rightarrow (2 \cdot 0.5, 10 \cdot 0.5) \rightarrow D'(1, 5)$$

Graph trapezoid  $A'B'C'D'$ .

To check the graph, draw lines from the origin through each of the vertices of the original figure. The vertices of the dilated figure should lie on the same lines.



Triangle  $LMN$  has vertices  $L(8, 2)$ ,  $M(10, 8)$ ,  $N(4, 6)$ . Find the coordinates of its image for a dilation with each given scale factor. Graph  $\triangle LMN$  and each dilation.

1. 0.5
  
2. 1.5
  
3. 2

