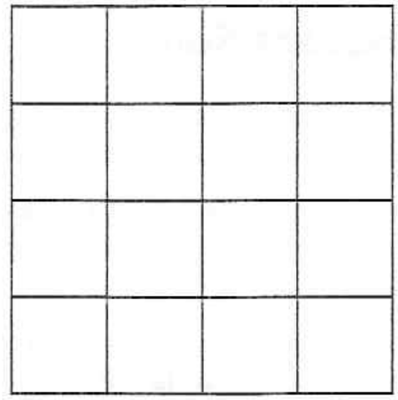
Name ______________________________ Date ______________

9-1 Study Guide

Square Roots



The area of a square is equal to the square of the length of its side. For the square shown, $A = 4^2$ or 16.

The length of a side of a square is equal to the square root of the area. For the square shown, 4 is the square root of 16.

If $x^2 = y$, then x is a **square root** of y. The symbol $\sqrt{\ }$ is called a **radical sign.** Read $\sqrt{16}$ as "the square root of 16."

Examples

x	$x^2 = y$	$\sqrt{y} = x$	
4	$4 \times 4 = 4^2 = 16$	$\sqrt{16} = 4$	*principal square root*
-4	$-4 \times -4 = (-4)^2 = 16$	$-\sqrt{16} = -4$	*negative square root*
1.5	$1.5^2 = 2.25$	$\sqrt{2.25} = 1.5$	
−1.5	$(-1.5)^2 = 2.25$	$-\sqrt{2.25} = -1.5$	

To find a square root, think of a number that when multiplied by itself (squared) equals the number under the radical sign.

Examples **Find each square root.**

$\sqrt{\frac{49}{64}} \longrightarrow \frac{7}{8} \times \frac{7}{8} = \frac{49}{64} \longrightarrow \sqrt{\frac{49}{64}} = \frac{7}{8}$

$\sqrt{-0.25} \longrightarrow (-0.5)^2 = 0.25 \longrightarrow -\sqrt{0.25} = -0.5$

Find each square root.

1. $\sqrt{100}$ **2.** $-\sqrt{144}$ **3.** $\sqrt{81}$ **4.** $-\sqrt{0.64}$

5. $\sqrt{\frac{9}{16}}$ **6.** $-\sqrt{6.25}$ **7.** $\sqrt{169}$ **8.** $-\sqrt{25}$

9. $-\sqrt{121}$ **10.** $\sqrt{\frac{25}{81}}$ **11.** $-\sqrt{0.16}$ **12.** $\sqrt{225}$

13. $-\sqrt{10{,}000}$ **14.** $\sqrt{400}$ **15.** $-\sqrt{1.21}$ **16.** $\sqrt{1.44}$

Name ______________________ Date ____________

9-2 Study Guide

Estimating Square Roots

Many numbers are not perfect squares. You can estimate square roots for these numbers.

Examples *1* Estimate $\sqrt{200}$.

$14^2 = 196$ *196 is a perfect square.*
$15^2 = 225$ *225 is a perfect square.*

$196 < 200 < 225$ *200 is between 196 and 225.*
$14^2 < 200 < 15^2$

$14 < \sqrt{200} < 15$ *The square root of 200 is between 14 and 15.*

Since 200 is closer to 196 than to 225, the best whole number estimate for $\sqrt{200}$ is 14.

2 Estimate $\sqrt{83.2}$.

$9^2 = 81$
$10^2 = 100$

$81 < 83.2 < 100$ *83.2 is between 81 and 100.*
$9^2 < 83.2 < 10^2$
$9 < \sqrt{83.2} < 10$

Since 83.2 is closer to 81 than 100, the best whole number estimate for $\sqrt{83.2}$ is 9.

Estimate to the nearest whole number.

1. $\sqrt{79}$ **2.** $\sqrt{24}$ **3.** $\sqrt{38}$ **4.** $\sqrt{103}$

5. $\sqrt{230}$ **6.** $\sqrt{85}$ **7.** $\sqrt{898}$ **8.** $\sqrt{1610}$

9. $\sqrt{15.5}$ **10.** $\sqrt{34.9}$ **11.** $\sqrt{50.6}$ **12.** $\sqrt{170}$

Name ______________________________ Date ______________

9-3 Study Guide

The Real Number System

Numbers may be classified into the following sets.

Natural Numbers	$\{1, 2, 3, 4, \ldots\}$
Whole Numbers	$\{0, 1, 2, 3, 4, \ldots\}$
Integers	$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
Rational Numbers	{numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$}
Irrational Numbers	{numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$}
Real Numbers	{rational numbers and irrational numbers}

Examples **Classify each real number.**

1 **0** whole number, integer, rational number

2 **0.777 . . .** Terminating or repeating decimals are rational numbers, since they can be expressed as fractions. $0.777\ldots = \frac{7}{9}$

3 $\sqrt{0.16}$ Square roots of perfect squares are rational numbers. $\sqrt{0.16} = 0.4$, a rational number.

4 $\sqrt{19}$ Square roots of numbers that are not perfect squares may be represented by decimals that do not repeat or terminate. They are irrational numbers.

To solve equations with squares, use your calculator to find square roots.

Examples **Solve each equation.**

5 $c^2 = 100$
$c = \sqrt{100}$ or $c = -\sqrt{100}$
$c = 10$ or $c = -10$

6 $x^2 = 60$
$x = \sqrt{60}$ or $x = -\sqrt{60}$
60 [2nd] [√] 7.745966692
$x = 7.7$ or $x = -7.7$

Name the set or sets of numbers to which each real number belongs.

1. 23 **2.** $\frac{7}{8}$ **3.** $\sqrt{31}$ **4.** 0.2727272. . .

5. $\frac{5}{11}$ **6.** 0.12131415. . . **7.** $\sqrt{0.81}$ **8.** -3

Solve each equation. Round solutions to the nearest tenth.

9. $m^2 = 81$ **10.** $x^2 = 5$ **11.** $t^2 = 0.49$ **12.** $n^2 = 1{,}600$

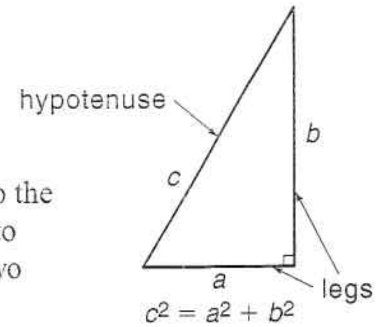
Name ______________________ Date ____________

9-4

Study Guide

The Pythagorean Theorem

The longest side of a right triangle is the **hypotenuse.** The hypotenuse is the side opposite the right angle. The other two sides of the triangle are the **legs.**



$c^2 = a^2 + b^2$

The **Pythagorean Theorem** relates the lengths of the sides of a right triangle: For any right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs. You can use the Pythagorean Theorem to find the length of a side of a right triangle if the lengths of the other two sides are known.

Examples

1 If $a = 30$ and $b = 40$, find c.

$c^2 = a^2 + b^2$
$c^2 = 30^2 + 40^2$
$c^2 = 900 + 1{,}600$
$c^2 = 2{,}500$
$c = \sqrt{2{,}500}$
$c = 50$

The length of the hypotenuse is 50 units.

2 If $c = 20$ and $a = 15$, find b.

$c^2 = a^2 + b^2$
$20^2 = 15^2 + b^2$
$400 = 225 + b^2$
$400 - 225 = b^2$
$175 = b^2$
$\sqrt{175} = b$
$13.228757 = b$

The length of the leg is 13.2 units.

The converse of the Pythagorean Theorem can be used to test whether a triangle is a right triangle: If the sides of a triangle have lengths a, b, and c units such that $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Find the missing measure for each right triangle. Round decimal answers to the nearest tenth.

1. $a = 8$ m; $c = 10$ m

2. $a = 5$ ft, $b = 12$ ft

3. $b = 15$ cm, $c = 25$ cm

4. $a = 7$ km, $c = 12$ km

5. $a = 8$ yd, $b = 11$ yd

6. $b = 14$ in., $c = 20$ in.

Determine whether each triangle with sides of given lengths is a right triangle.

7. 20, 21, 29

8. 7, 24, 25

9. 9, 11, 14

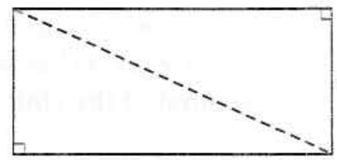
Name ____________________ Date ____________

9-5 Study Guide

Using the Pythagorean Theorem

You can use the Pythagorean Theorem to help you solve problems.

Example **An ice hockey rink is 200 feet long and 85 feet wide. What is the length of the diagonal of the rink?**



$c^2 = a^2 + b^2$
$c^2 = 200^2 + 85^2$
$c^2 = 40{,}000 + 7{,}225$
$c^2 = 47{,}225$
$c = \sqrt{47{,}225}$
$c \approx 217.3$

The length of the diagonal of an ice hockey rink is about 217.3 feet.

The chart shows some Pythagorean triples, numbers that satisfy the Pythagorean Theorem. You can find other Pythagorean triples by finding multiples of these triples.

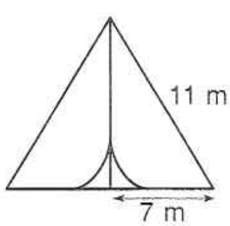
a	*b*	*c*
3	4	5
5	12	13
8	15	17
9	40	41

	a	*b*	*c*
original	5	12	13
× 2	10	24	26
× 3	15	36	39
× 4	20	48	52

10-24-26, 15-36-39, and 20-48-52 are members of the 5-12-13 family.

Solve. Round answers to the nearest tenth.

1.

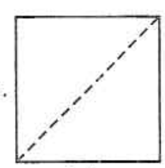


What is the height of the center pole?

2.

What is the length of the ramp?

3.



What is the length of the diagonal?

Name ______________________ Date __________

9-6 Study Guide

Distance on the Coordinate Plane

You can use the Pythagorean Theorem to find the distance between two points on the coordinate plane.

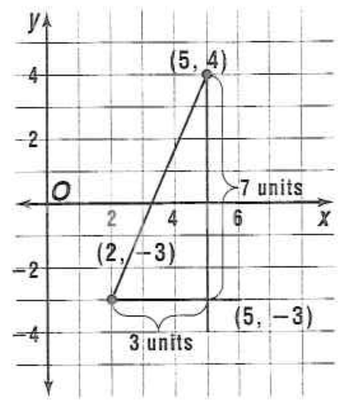
Example **Find the distance between the points at (2, −3) and (5, 4).**

Graph the points and connect them with a line segment. Draw a horizontal line through (2, −3) and a vertical line through (5, 4). The lines intersect at (5, −3).

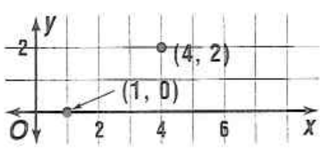
Count units to find the length of each leg of the triangle. Then use the Pythagorean Theorem to find the hypotenuse.

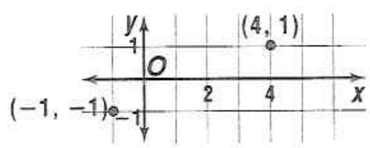
$c^2 = a^2 + b^2$
$c^2 = 3^2 + 7^2$
$c^2 = 9 + 49$
$c^2 = 58$
$c = \sqrt{58}$
$c \approx 7.6$

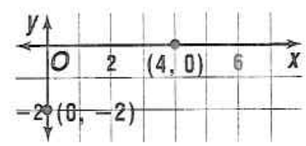
The distance between (2, −3) and (5, 4) is about 7.6 units.



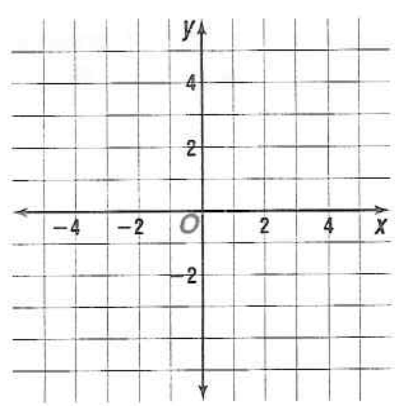
Find the distance between each pair of points whose coordinates are given. Round to the nearest tenth.

1. 

2. 

3. 

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth.



4. (4, 5); (0, 2)

5. (0, −4); (−3, 0)

6. (3, 1); (1, −4)

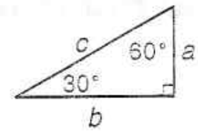
7. (−1, 1); (−4, 4)

Name ______________________ Date ____________

9-7 Study Guide

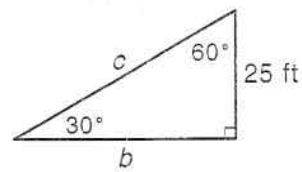
Integration: Geometry
Special Right Triangles

In a 30°-60° right triangle, the length of the side opposite the 30° angle is one-half the length of the hypotenuse. You can use this relationship to solve problems.



$a = \frac{1}{2}c$ or $c = 2a$

Example 1 **Find the lengths *b* and *c*.**



Find *c*.

$c = 2a$

$c = 2(25)$

$c = 50$

Find *b*.

$c^2 = a^2 + b^2$

$50^2 = 25^2 + b^2$

$2{,}500 = 625 + b^2$

$2{,}500 - 625 = b^2$

$1{,}875 = b^2$

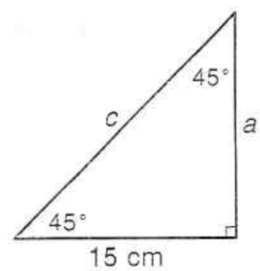
$\sqrt{1{,}875} = b$

$43.3 \approx b$

The length of hypotenuse *c* is 50 feet.
The length of side *b* is about 43.3 feet.

In a 45°-45° right triangle, the lengths of the legs are equal. You can use this relationship to solve problems.

Example 2 **Find the lengths of *a* and *c*.**



Find *a*.

$a = b$

$a = 15$

Find *c*.

$c^2 = a^2 + b^2$

$c^2 = 15^2 + 15^2$

$c^2 = 225 + 225$

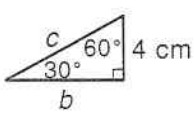
$c^2 = 450$

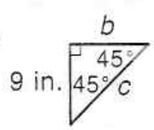
$c = \sqrt{450}$

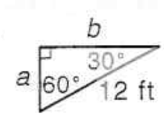
$c \approx 21.2$

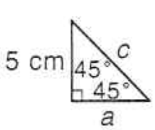
The length of side *a* is 15 cm.
The length of hypotenuse *c* is about 21.2 cm.

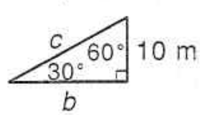
Find the missing lengths. Round decimal answers to the nearest tenth.

1. 

2. 

3. 

4. 

5. 

6.