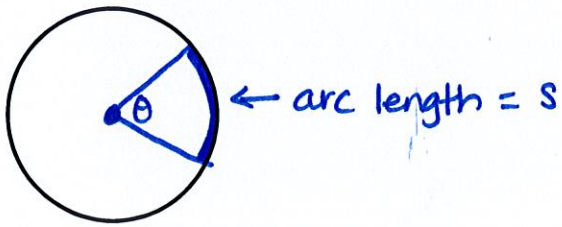
**Pre-Calculus** **Arc Length, Linear/Angular Velocity Notes** **Name:**

**Arc Length:**

part of circumference

$\frac{\theta}{2\pi} \cdot 2\pi r = r\theta$

↳ $\theta$ must be in radians



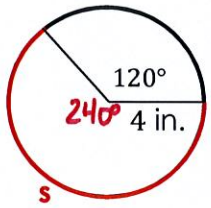

**Example 1:** A circle has a radius of 4 inches. Find the length of the arc intercepted by a central angle of 240°.

$r\theta = 4 \cdot 240^\circ \cdot \frac{\pi}{180^\circ} =$ 16.76 in

**Example 2:** Find ***s***

Same as Ex. 1!

Need to recognize 240° is θ




**Example 3:** Find the central angle (in radians and degrees) that is formed by a radius of 12 ft and an arc length of 31 ft

$S = r\theta$

$31 = 12\theta$

$\theta = \frac{31}{12} =$ 2.58 rad.

$2.58 \cdot \frac{180^\circ}{\pi} \approx 147.8^\circ$

**Example 4:** Find the **distance between the cities**. Assume that the Earth is a sphere of radius 3960 miles and the cities are on the same longitude (one city is due north of the other).

| | |
|---|---|
| Johannesburg, South Africa | 26° S |
| Jerusalem, Israel | 31° N |

$\theta = 26^\circ + 31^\circ = 57^\circ$


$S = r\theta = 3960 \cdot 57^\circ \cdot \frac{\pi}{180^\circ} \approx$ 3939.56 mi.

**Write a formula relating distance, rate and time:** D=RT

**Linear Speed:** * speed at which something moves around a circular figure

**Angular Speed:** * speed at which an angle moves around a circular figure

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**Equivalent ratios:** * Circumference = $2\pi$ rad = 360° = 1 revolution ($2\pi r$ units)

**Equivalent ratios:** $2\pi$ radians = $360°$ = 1 revolution = circumference [$2\pi r$ "units"]

**Example 5:** The second hand of a clock is 10.2 cm long. Find the linear speed of the tip of the second hand in cm/s.

$$\frac{\text{circumference}}{\text{time}} = \frac{2\pi \cdot 10.2 \text{ cm}}{60 \text{ sec}} = \boxed{1.07 \text{ cm/sec.}}$$

**Example 6:** A 15- inch diameter tire on a car makes 9.3 revolutions per second.

**a.** Find the angular speed of the tire in rad/sec

$$\frac{9.3 \text{ rev.}}{\text{sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \boxed{58.43 \text{ rad/sec}}$$

**b.** Find the linear speed of the car in in/sec

$$\frac{2 \cdot \pi \cdot 7.5 \text{ in}}{1 \text{ rev.}} \cdot \frac{9.3 \text{ rev.}}{\text{sec}} = \boxed{438.25 \text{ in/sec}}$$

★ units have to cancel to be left with rad/sec. (dimensional analysis)

**Example 7:** The circular blade on a saw has a diameter of 7.25 inches and rotates at 4800 revolutions per minute.

**a.** Find the angular speed of the blade in rad/sec

$$\frac{4800 \text{ rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev.}} = \boxed{502.65 \text{ rad/sec}}$$

**b.** Find the linear speed of the saw teeth (in ft/sec) as they contact the wood being cut.

$$\frac{4800 \text{ rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec.}} \cdot \frac{7.25\pi \text{ in}}{1 \text{ rev.}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} =$$

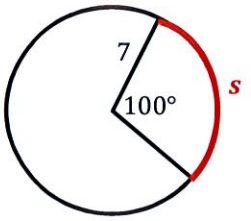
$$\boxed{151.84 \text{ ft/sec}}$$

**Example 8:** A woman is riding a bicycle whose wheels are 30 inches in diameter. If the wheels rotate at 150 rpm, find the speed at which she is traveling in **mi/hr**.

$$\frac{150 \text{ rev}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr.}} \cdot \frac{30\pi \text{ in}}{1 \text{ rev.}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} = \boxed{13.39 \text{ mi/hr}}$$

**Pre-Calculus** **Arc Length, Linear/Angular Velocity Homework Worksheet**

**1.** Find the length of the arc $s$ in the figure.

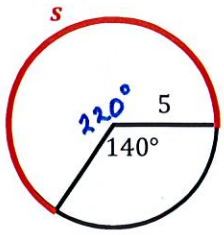



$S = r\theta$

$= 7 \cdot 100° \cdot \frac{\pi}{180°}$

$S = 12.22$ units

**2.** Find the length of arc $s$ in the figure.




$S = 5 \cdot 220° \cdot \frac{\pi}{180°}$

$S = 19.20$ units

**3.** Find the length of an arc that subtends [forms] a central angle of 45° in a circle of radius 10 m.


$S = r\theta = 10 \cdot \frac{\pi}{4} = 7.85\text{m}$

**4.** Find the length of an arc that subtends [forms] a central angle of 2 rad in a circle of radius 2 mi.

$S = r\theta = 2 \cdot 2 = 4$ mi

**5.** An arc of length 100 m subtends [forms] a central angle $\theta$ in a circle of radius 50 m. Find the measure of $\theta$ in radians and degrees.

$S = r\theta$

$100 = 50\theta$

$\theta = 2$ rad.

$2 \text{ rad} \cdot \frac{180°}{\pi \text{ rad}} = 114.6°$

**6.** Find the radius of the circle if an arc of length 6 m on the circle subtends [forms] a central angle of $\frac{\pi}{6}$ rad.

$6 = r \cdot \frac{\pi}{6}$

$r = \frac{36}{\pi} = 11.46$ m

**7.** Memphis, TN and New Orleans, LA lie approximately on the same meridian (longitude line). Memphis has a latitude 35°N and New Orleans has a latitude 30°N. Find the distance between these cities if the radius of the earth is 3960 mi.

$\theta = 5°$

$S = r\theta = 3960 \cdot 5° \cdot \frac{\pi}{180°}$

$S = 345.58$ mi

**8.** A radial saw has a blade with a 6-in radius. Suppose the blade spins at 1000 rpm.

**a)** Find the angular speed of the blade in rad/min.

$\frac{1000 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 6283.19$ rad/min

**b)** Find the linear speed of the saw teeth in ft/sec.

$\frac{1000 \text{ rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{2\pi(6) \text{ in}}{1 \text{ rev}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 52.36$ ft/sec

**9.** The wheels of a car have a diameter of 22 in and are rotating at 600 rpm. Find the speed of the car in mi/hr.

$$\frac{600 \text{ rev}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{2\pi(11) \text{ in}}{1 \text{ rev}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} = 39.27 \text{ mph}$$

**10.** The earth rotates about its axis once every 23 h 56 min 4 s, and the radius of the earth is 3960 mi. Find the linear speed of a point on the equator in mi/hr.

$$\frac{2\pi \cdot 3960 \text{ mi}}{23.9\overline{3} \text{ hr}} = 1039.57 \text{ mi/hr}$$

convert time to only hours!

**11.** A wind machine used to generate electricity has blades that are 10 ft in length. The propeller is rotating at 4 revolutions per second. Find the linear speed of the tips of the blades in ft/min.

$$\frac{4 \text{ rev}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{2\pi(10) \text{ ft}}{1 \text{ rev}} = 15079.64 \text{ ft/min}$$

**12.** The carousel at the county fair makes 3 revolutions per minute.

**a)** Find the linear speed in ft/sec of a person riding a horse that is 22.5 ft from the center.

$$\frac{3 \text{ rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{2\pi(22.5) \text{ ft}}{1 \text{ rev}} = 7.07 \text{ ft/sec}$$

**b)** The linear speed of the person on the inside of the carousel is 3.1 ft/sec. How far is this person from the center?

$$3.1 \frac{\text{ft}}{\text{sec}} = \frac{3 \text{ rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{2\pi x \text{ ft}}{1 \text{ rev}} \qquad x = 9.87 \text{ ft}$$

**c)** How much faster is the rider on the outside going than the rider on the inside?

$$7.07 \frac{\text{ft}}{\text{sec}} - 3.1 \frac{\text{ft}}{\text{sec}} = 3.97 \frac{\text{ft}}{\text{sec}}$$

**Answers:**

**1.** 12.22 units **2.** 19.20 units **3.** 7.85 m **4.** 4 mi **5.** $\theta = 2$ rad or $\theta = 114.6°$

**6.** $r = \frac{36}{\pi}$ m or $r = 11.46$ m **7.** 345.6 mi **8a.** $2000\pi$ rad/min or 6283.19 rad/min **8b.** 52.36 ft/sec

**9.** 39.27 mph **10.** 1039.6 mph **11.** 15079.64 ft/min **12a.** 7.07 ft/sec **12b.** 9.87 ft **12c.** 3.97 ft/sec