IPMG Publishing

For Grades 2-12
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Classroom Tested A+ Teacher Approved

Math Games and Activities with Dice

by William L. Gaslin, Charles Lund, Martin M. Gaslin

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IPMG Publishing
18362 Erin Bay
Eden Prairie, Minnesota 55347

phone: (612) 802-9090

www.iplaymathgames.com

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Introduction

Dice have been used as a recreational pastime for thousands of years. One of the earliest examples is the four-sided dice from Egyptian tombs. Etruscan dice, found near Rome, and made about 900 B.C.E., are similar to the dice of today.

During the 12th and 13th centuries, dice spread throughout England and were often played by men in taverns for almost anything, even their clothing.

Because of the link to gambling, dice games met with strong disapproval by many of the early colonists in America. Despite these admonitions, dice still became a common household item in the North American colonies and have remained a part of the standard gaming inventory in most homes ever since.

Many dice games have been passed down through the centuries. A few of the games have even been modified and marketed by commercial vendors. Unfortunately, few resources are available to provide a link between this powerful motivational tool and mathematics instruction in Grades 2-8. That is the purpose of this resource book. It is our hope that you and your students will not only be able to relive the excitement of playing dice games, but that they will also sharpen their math skills. The games and activities usually stimulate a discussion as a follow up to each session. These discussions are important because they enhance students' vocabulary development, speaking skills, links to the math content and provide opportunities to suggest improvements to a game or to design a new game. Most of the material can be copied and used immediately. Some games are generic in nature and designed to be modified by teachers and/or students. The games and activities are also useful ideas to send home to families or as summer fun. If some parents are concerned about the potential link to gambling, encourage them to refer to the dice as number cubes.

The table of contents groups the games and activities by topic. Within each topic the games are organized alphabetically, but not sequentially. Teacher comments and selected answers are also provided.

Why use dice?

Dice are:
A useful diagnostic tool
Highly motivating
Flexible! They can be used at numerous grade levels and in many settings
Inexpensive and readily available
A powerful springboard to new topics based on pupil experience
Helpful as an aid to reinforce concepts
Effective as a tool to provide skill maintenance
A familiar, tactile, visual, yet mysterious and magical way to provide variety

When and where to use dice:

As a class starter.
Friday afternoon.
Special days and events.
Day before holiday.
As a "sponge" or "filler."
Whenever the objective warrants.
Individually.
One on one.
In small groups.
As an entire class.
As an activity station.

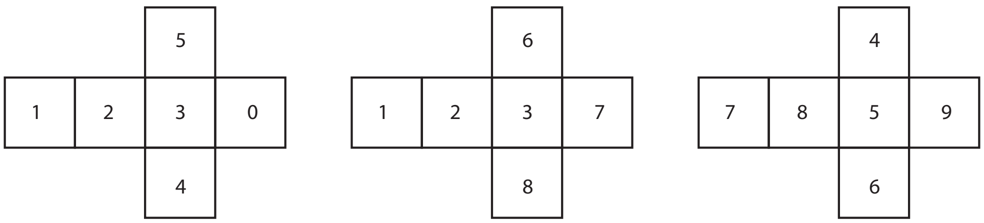
Management tips for using dice games:

Make sure the game fits your objectives.
Explain the rules before play using transparent dice or "giant" dice.
Establish ground rules before play begins.
Keep the group size at four or less.
Match students of comparable ability in competitive games.
Be sure to provide a follow up discussion for each game. The discussion can include sharing strategies to win, questions and what was learned from the game.
If parents are concerned about the link to gambling, send a sample game home with a note explaining the objective of the game.
Use the games sparingly.

Around The Schoolyard

Materials

Three customized dice labeled as shown below for each group.



One *Around the Schoolyard* game board for each player/team.
One marker for each player/team.

Rules and Play

1. This is a division of whole numbers with remainders dice game for 2-4 players/teams. The object of the game is to be the first player/team to go all around the schoolyard by solving problems. Players must travel from start to finish, counterclockwise, one station at a time.

2. To start the game, each player/team places a marker at the start. Next, the first player/team rolls all 3 dice. In every dice roll, 3 1-digit numbers will appear. The player/team must then select 1 number to be the divisor. The remaining numbers are used to form the dividend. For example, if the numbers were 8, 3 and 1, they could be arranged to form 18/3. If the quotient (6) is found in the next area of the schoolyard, the player/team advances the marker. If the desired answer has a remainder, the player/team gets one try to roll the 3 dice to come up with that number.

3. Players/teams continue taking turns. Each answer must be in the form AB/C. Zero must not be used as the divisor, and only the numbers on the dice are to be used to obtain an answer.

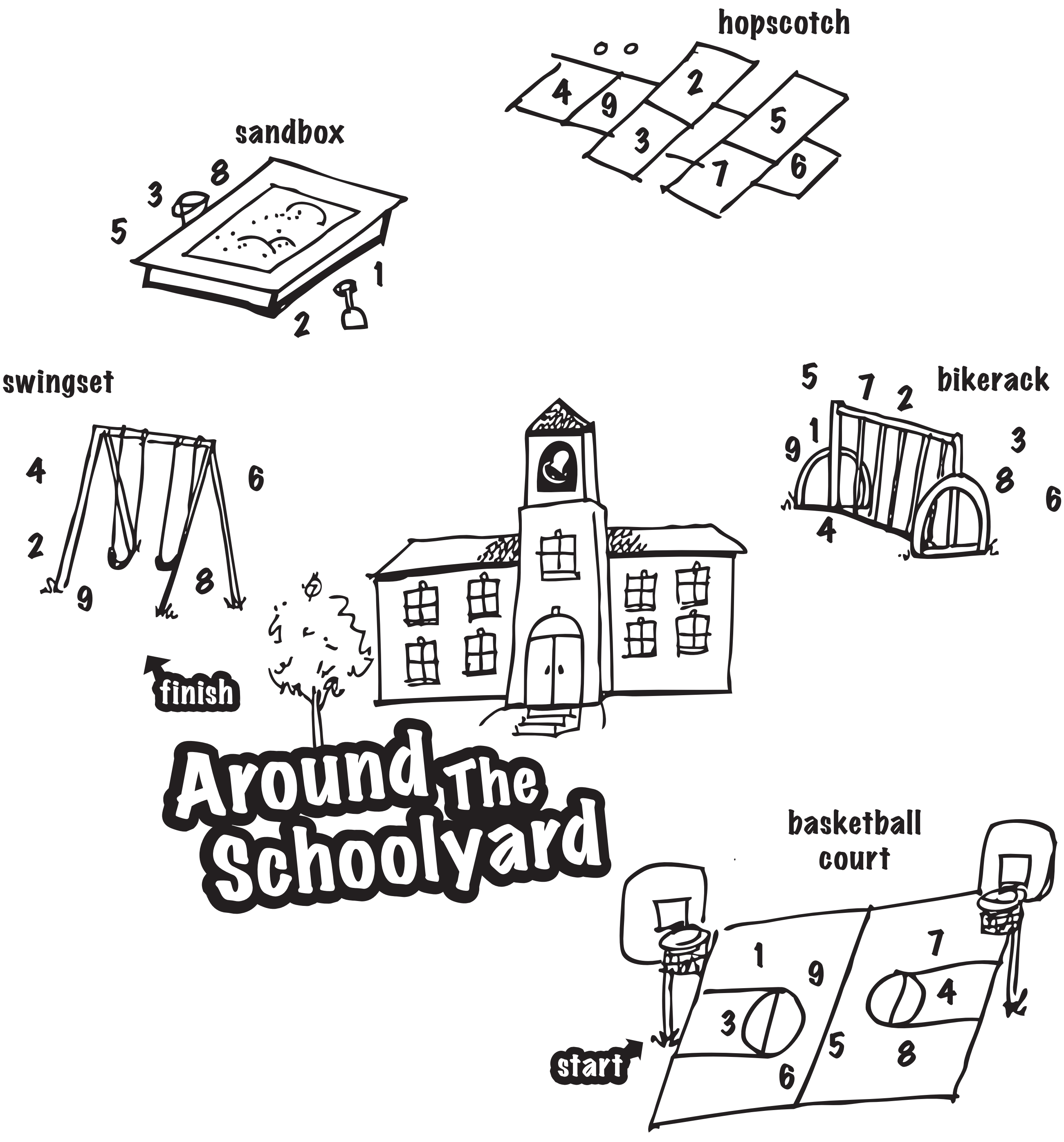
4. The player/team that circles the schoolyard first wins.

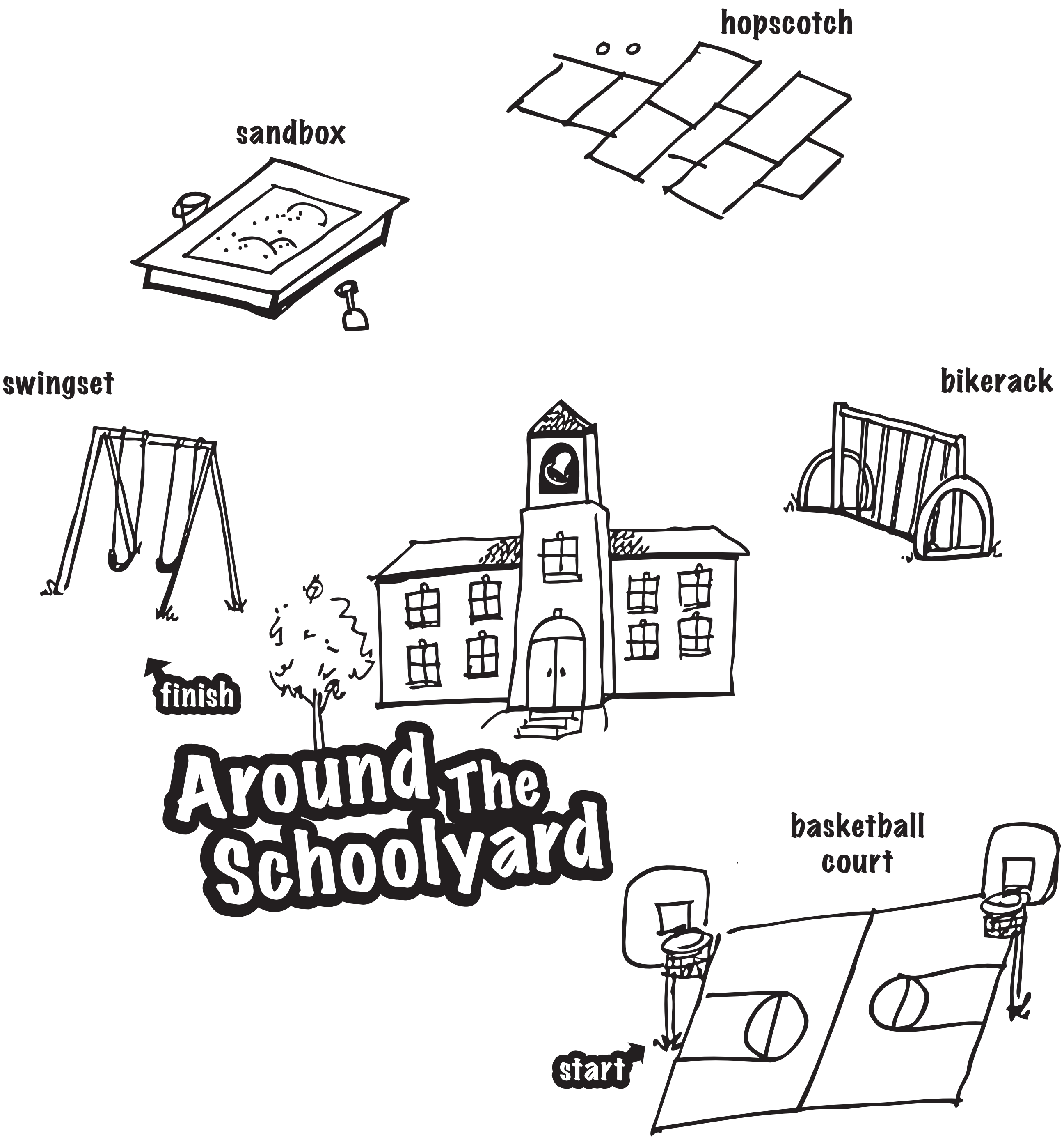
Variations

- Change the numbers on the dice and playing board using the generic *Around the Schoolyard* game board master.

- Adapt the game to addition, subtraction and multiplication by using 2 customized dice and the generic *Around the Schoolyard* game board.

- Redesign the game board to an interdisciplinary setting such as *Tour The Planets*.


hopscotch
4 9 2 3 5 7 6
sandbox
5 3 8 1 2
swingset
4 6 2 9 8
bikerack
5 7 2 1 9 3 8 6 4
finish
Around The Schoolyard
basketball court
1 9 7 3 4 5 8 6
start


hopscotch
sandbox
swingset
bikerack
finish
Around The Schoolyard
basketball court
start

COUNT:DOWN

Materials

Two dice for each group. The dice may be standard or custom made.

Rules and Play

1. This is a subtraction of whole numbers game for 2-4 players/teams. The object of the game is to be the first player/team to reach or go below zero.

2. Each player/team starts the game with 99 points (or whatever number desired). Players/teams take turns rolling the dice, finding the dice sums and then subtracting the total from their score. After computing the results, the score is recorded and the dice pass to the next player/team.

3. Play continues until one player/team reaches or goes below zero.

Variations

- Change the goal so that the object is to come as close to zero as possible without going below it. A player/team who goes below zero is out of the game.

- For young students, use one die. Start at 29 so that the first roll does not require regrouping.

- Use other dice patterns.

- Use any starting and stopping point. For example, start at 999 and stop at 700.

- Play using a calculator.

- To make the game more challenging, require players to finish exactly at zero.

- Use each die to represent a place value location. For example, if 2 dice were used and a 5 and a 7 were tossed, a player could subtract 57 or 75 from his/her score.

- Vary the number of dice. For example, use 3 dice, 1 die for ones, 1 die for tens, and 1 die for hundreds. Start the game at 2999.

- Adjust the numerals on the dice to include decimals, fractions or integers.

- Limit the number of turns for a game. For example, the player/team closest to zero after 7 tries is the winner.

COVER UP

Materials

One playing board for each pair of players.
Two standard dice and counters for each pair of players.
Note that the teacher must design the playing board so that it is appropriate for the dice (s)he wishes to use.
For example, if a teacher used 2 standard dice, and the operation of addition, then the board would be labeled as shown.

Rules and Play

1. This is a dice game for 2 players.

2. Players sit on opposite side of the playing board. The *Cover Up* game board consists of 2 rows of numbered squares. Players take turns rolling the dice and performing the desired operation on the 2 numbers that appear. If the answer is correct, the cell with the answer on the player's side of the board is covered. If a cell is already covered, or an incorrect answer is given, the player must pass. The winner is the player with the greatest number of covered squares at the end of 10 rounds. If both players cover up the same number of squares, the player with the largest number of correct answers wins. If those numbers are the same, the game is tied. The time limit for each problem is 30 seconds.

Free	12	11	10	9	8	7	6	5	4	3	2
COVER UP GAME BOARD											
2	3	4	5	6	7	8	9	10	11	12	Free

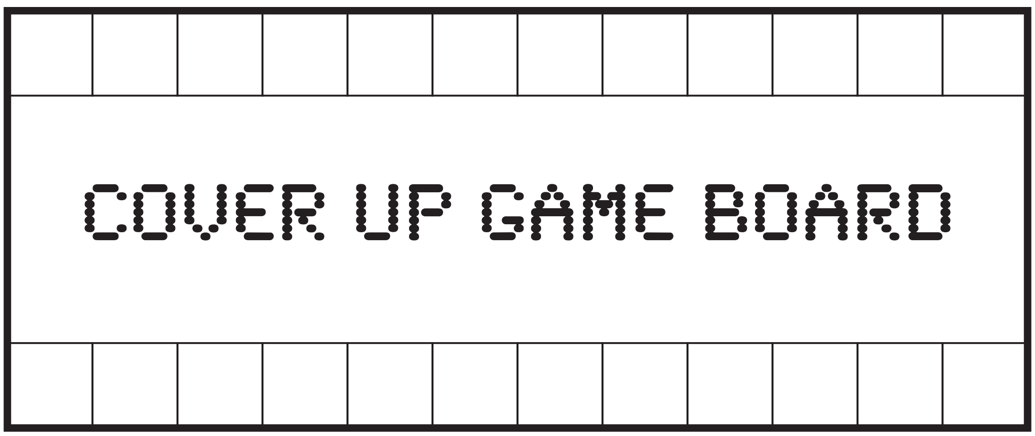
Variations

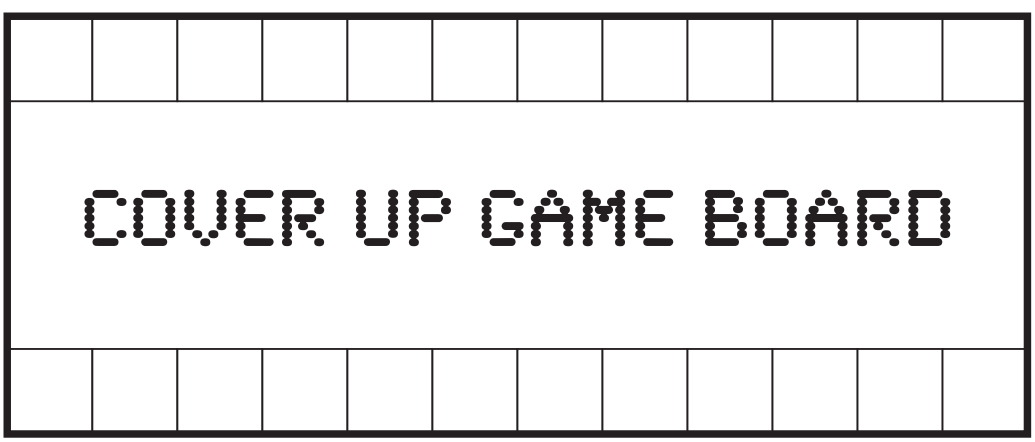
- Use more than 2 standard dice.
- Use different sets of whole numbers.
- Shorten, extend or eliminate the time limit for each problem.
- Change the number of rounds in a game or eliminate the "Free" space.

COVER UP

Variations

Use blank *Cover Up* game boards to create your own game.





Selected Answers and Comments

Page 44: Power Play. Suggested follow up questions are provided below.

1. What is the largest score possible in one round of a game of Power Play if two ordinary dice are used? Answer: 66 = 46,656.
2. What is the smallest score possible in one round of a game of Power Play if two ordinary dice are used? Answer: One to any power (e.g. 16 = 1).
3. What is the second largest score possible in one round of a game of Power Play if two ordinary dice are used? Answer: 56 = 15,625.
4. What is the second smallest score possible in one round of a game of Power Play if two ordinary dice are used? Answer: 21 = 2.
5. What is the highest total score possible in an eight-round game of Power Play using two ordinary dice? Answer: 46,656 x 8 = 373,248.

Page 45: Block Builder. 2a. 10 2b. 100 2c. 10 2d. 100 2e. 100 2f. 10 To save time and encourage cooperation, consider having pairs of students work together to build a cubic decimeter. Have each group tally the number of turns needed to complete the task. Also, consider awarding a small prize to the group who completes the job in the fewest number of turns.

What is the average number of turns needed to complete a game in our class?

What is the minimum number of turns needed to build a cubic decimeter using a die labeled 0, 5, 10, 25, 50, 100? Answer: 10

What is the maximum number of turns needed to build a cubic decimeter using a die labeled 0, 5, 10, 25, 50, 100? Answer: Infinite

What is the theoretical average number of rolls that should be needed to build a cubic decimeter using a die labeled 0, 5, 10, 25, 50, 100?
Answer: $[5 + 10 + 25 + 50 + 100] / 6 \approx 31.67$, $1000/31.67 \approx 31.58$, so, 32 turns.

Similar questions can be posed using ordinary dice.

Page 51: One Meter Dash. To save time and encourage cooperation, consider having pairs of students work together to build a cubic decimeter. Have each group tally the number of turns needed to complete the task. Also, consider awarding a small prize to the group who completes the job in the fewest number of turns.

What is the average number of turns needed to complete a game in our class?

What is the minimum number of turns needed to build a cubic decimeter using a die labeled 0, 2, 10, 1, 5, -1? Answer: 10

What is the maximum number of turns needed to build a cubic decimeter using a die labeled 0, 2, 10, 1, 5, -1? Answer: Infinite

What is the theoretical average number of rolls that should be needed to build a cubic decimeter using a die labeled 0, 2, 10, 1, 5, -1?
Answer: $[0 + 2 + 10 + 1 + 5 + -1] / 6 \approx 2.83$, $100/2.83 = 35.34$, so, 36 turns.

Similar questions can be posed using ordinary dice.

Page 52-53: It's Your Call. 2. .125 x 30 = $3.75 3. Lucky. Extra Credit: The expected earnings for this game can be calculated as outlined below. Note: The probability of one five on each of the dice is 1/6. So, the probability of getting one five on the 3 dice is:

First Die 5	Second Die not 5	Third Die not 5	
1/6	5/6	5/6	= 25/216
First Die not 5	Second Die 5	Third Die not 5	
5/6	1/6	5/6	= 25/216
First Die not 5	Second Die not 5	Third Die 5	
5/6	5/6	1/6	= 25/216

The probability of one five on the three dice is...75/216

Selected Answers and Comments

First Die 5	Second Die 5	Third Die not 5	
1/6	1/6	5/6	= 5/216
First Die 5	Second Die not 5	Third Die 5	
1/6	5/6	1/6	= 5/216
First Die not 5	Second Die 5	Third Die 5	
5/6	1/6	1/6	= 5/216

The probability of not getting two fives on the three dice is...15/216

The probability of three fives on the 3 dice is:
1/6 x 1/6 x 5/6 = 1/216.

So, the expected earnings for Player 1 are:
75/216($1) + 15/216($2) + 1/216($3) = 96/216 or .44

The expected earnings for Player 2 are:
31/216(-1) = -31/216 = -.1435 or -.14

Page 60: 654. Sample results from 3 sets of 20 games are provided.

W	L	W	L	W	L
|||	卌 卌 卌	卌 ||	卌 卌 卌	||||	卌 卌 卌 |

1a. No. Explanations will vary. 1b. Yes. Explanations will vary.

Pages 55-56: 3 Ways to Win. 2a. $30. 2b. Answers will vary. An example is shown.

Result	Frequency	Payoff	Total
1	11	0	0
2	5	0	0
3	12	.20	2.40
4	13	.30	3.90
5	11	1	11
6	8	.20	1.60

Sum: 18.70

3e. $E = 1\left(\frac{1}{6}\right) + .30\left(\frac{1}{6}\right) + .20\left(\frac{2}{6}\right)$

$$E = \frac{1+.3+.4}{6} = \frac{1.7}{6} \approx .28\overline{3}$$

3f. No. The expected payoff for a fair game is zero. In this game it is .283 - .50 is approximately equal to -.22
4. Yes. Charge less to game play. .283 is the expected value, so charging .28 would be close.

Pages 57-58: Two Dice Sums. 1a. $72 2a. $54 2b. $72 2c. -$22 3. No 4. Yes. Change the cost of playing. 56/72 = .78. If we find the product of each outcome and the probability of that outcome and add the results, we get

$$\left[\left(\tfrac{2}{36}\right) \times 10\right] + \left[1 \times \left(\tfrac{2}{36}\right)\right] + \left[\left(\tfrac{1}{2}\right) \times \left(\tfrac{14}{36}\right)\right] = \left(\tfrac{28}{36}\right) \approx .78$$

5. $E = \left[\frac{4}{36}(0) + \frac{3}{36}(0) + \frac{2}{36}(0) + \frac{1}{36}(0)\right] - \frac{1}{(1)} = \frac{28}{36} - 1 = \frac{7}{9} - 1 = \frac{-2}{9}$ or $-.\overline{2}$

Page 59: Dice Calendar Puzzle. Answers will vary. One example is shown here. Note that the numerals 6 and 9 are shown by rotating the cube.

		5	
0	1	2	3
		4	

		6	
1	2	0	7
		8	

Selected Answers and Comments

Page 60: Stacking Dice Puzzles 1. 1,6 2,5 3,4 2. 7 3. 21 4. 42, 63, 84, 105 5. 210, 21000, 21000 6. 21n 7. Yes. 7 8. No. 9. Yes. 21 10.

Number of Dice in Stack	Sum of Each Side of Stack
2	7
3	Impossible
4	14
5	Impossible
6	21
7	Impossible

11a. Yes. 11b. 42 11c. 350 11d. 3500 11e. n/2 x 7

Page 62-63: Touching Faces Dice Puzzles. 1. Top face is a 4. Bottom face is a 3. Left face is a 6. Right face is a 5. 2a. 5+1=6. 2b. 6+4=10 2c. (3+2)=5 3a. 5+1+4=10 3b. 6+4+2=12 3c. (3+2) + (5+1) = 11 4a. 5+1+4+3=13 4b. 6+4+2+1=13 4c. (3+2) + (5+1) + (2+6) = 19 5. The key to solving each problem is knowing that the sum of the opposite faces on an ordinary die is 7.

Page 64: Rolling 3 Dice Trick. Let a, b, and c represent the values shown on the dice. Adding the numbers on the facts can be represented by: a + b + c.

One die is picked up and the number on the bottom face of that die is added to the previous sum. So, the number on the opposite face of a is 7-a, the number on the opposite face of b is 7-b, and the number on the opposite side of c is 7-c.

Suppose "b" is the number shown on the die picked up. Since the number on the opposite face of b is 7-b, the new total will be (a+b+c) + (7-b). Simplifying we get a+c+7 as the new total.

Since all three dice have the same characteristic number arrangement, the new sum will always be equal to 7 plus the values on the two dice not selected.

Rolling the selected die again gives another number to add to our total. Let's call it d. Note that d might have the same value as a, b, or c, but it doesn't matter. Adding the value d to running total we get: a+c+7+d.

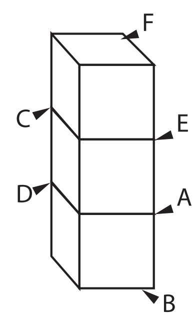
What you see on the table is a+c+d.

What your partner has totaled is a+c+d+7.

All you need to do is add 7 to what you see to predict your partner's secret total.

Teacher comment: Using stickers labeled a, b, c, and d on the 3 dice can be useful in explaining the proof to students.

Page 65: Touching Faces Dice Trick.



2.

Proof.

Recall that the sum of opposite faces of a die = 7.
So, we know that (a+b)=7, (c+d)=7, and (e+f)=7.
We also know that (a+b) + (c+d) + (e+f) = 21.

What we need to show is that (e+c) + (d+a) + b =21 – f is another way to write (a+b) + (c+d) + (e+f) = 21.

We can do this by starting with (a+b) + (c+d) + (e+f) = 21. Then, removing the parentheses we get a + b + c + d + e + f = 21. Next we change the order of the variables and get e + c + d + a + b + f = 21. Finally, we subtract f from both sides of the equation and get e + c + d + a + b = 21 – f. So, it will work all the time!

Teacher comment: Using stickers labeled a, b, c, d, e, and f on the 3 dice can be useful in explaining proof to students.