→ "The Distributive Property: The Core of Multiplication" appendix (page 1 of 5)

Activity 1: Go fish

This game is like the traditional Go Fish game, except that the goal is to put down sets of three cards that make an addition equation. The game may be played in grades 1–5.

Materials

- A standard deck of cards for each group of 2–4 students. (Be sure to remove the jokers.) Other kinds of numbered decks (like Uno cards) will also work.
- Counters, such as coins or tokens for younger children

Purpose

- To gain fluency in decomposing the numbers from 0–10
- To form a foundation for mental addition of a one-digit number to a two-digit number

Rules

- 1. The game is played by a group of 2–4 players. The first player to eliminate all his or her cards wins. If the deck runs out, the player with the fewest cards in his or her hand wins.
- 2. Shuffle the cards, and deal seven cards to each player. Place the remainder of the deck in a pile, facedown.
- 3. Players may look at their own cards but not at each others' cards.
- **4.** At the beginning or the end of their turn, players with three cards in their hand that make an addition sentence may place those three cards in a group, faceup on the table in front of them, as in the picture above.
- **5.** On your turn, choose another player, and ask for a card. For example, "Janice, do you have a king?" If Janice has a king, she must give it to you. If not, Janice says, "Go fish," and you draw a card from the deck.
- **6.** After you get a card, either from another player or from the deck, and put down any groups of three that you have, then your turn is over.
- 7. Aces have the value 1, and face cards all have the value 0 in number sentences.

Variations and scaffolding

- It is not always easy to spot a set of three cards that you can put down—even adults do not always see the sets at first. Do not push the children to put down their sets right away. Help them to develop strategies to find their own sets.
- For younger students, try starting with only the cards 1–5. You may want to use the cards from two decks.
- Do not put the zeros (face cards) in until students are comfortable with the other numbers (but see the remark about zero).
- For younger students, practice with everyone's cards faceup. This will allow students to discuss possible moves; it also frees their hands so that they can use their fingers or counters.
- Have older children say the equation when they put down a set of three, such as, "Four plus five equals nine," or "Eight plus zero equals eight."
- Allow more advanced players to put down more than three cards, as long as they can make them into an addition equation. For example, a player could put down 3, 4, 5, and 2 because 3 + 4 = 5 + 2. This makes the game more interesting and challenging, and reinforces the idea that a number sentence can have more than one number on both sides of the equation.



Activity 2: Adding up the deck

In this game, students play as a team to add up all the cards in a deck. Students have the opportunity to develop and share strategies for adding numbers. The game may be played in grades 2–5.

Materials

- A standard deck of cards for each team of 2–4 students. (Be sure to remove the jokers.) Other kinds of numbered decks (like Uno cards) will also work.
- A number line and a pencil for younger students

Purpose

Use knowledge of decomposing small numbers to develop strategies for mentally adding a one-digit number to a two-digit number. Practice both reasoning and computation with addition. This is a good game for students to discuss effective strategies for cooperating to help each other learn. On the one hand, students have an incentive to help one another avoid mistakes. On the other hand, the game is not fun if one student dominates and tells the others all the answers.

Rules

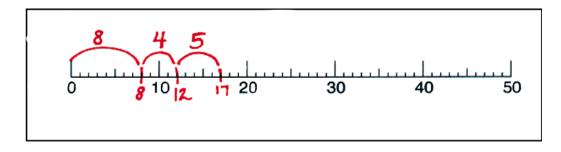
- 1. The game is played in teams of 2–4 players. Once students develop some fluency, the teams may compete against each other to see who can first finish the deck correctly.
- **2.** Aces have a value of 1, and all face cards have the value of 0. They still have to be played one at a time, in turn, like the other cards.
- 3. The dealer deals out all the cards to members of the team.
- 4. The first player begins by putting down a card and saying its value. The next player puts down a card and adds its value to the total so far. For example, the first player might put down an 8 and say, "Eight." Then the next player might put down a 4 and say, "Twelve." The third player might put down a 5 and say, "Seventeen." Students may keep track of the total with hops on a number line, as shown below.



Variations and scaffolding

Encourage students to use strategies for addition that make sense to them. Some students will at first be most comfortable using the number line or their fingers. As they play the game more times, they should be encouraged to use more efficient strategies based on decomposing numbers.

- For beginning students, use cards 1–5 at first (but see our remark about zero).
- Students are likely to make errors the first few times that they play. At some point, you can have a productive discussion about whether



- the total should always be the same in the end. This encourages students to own the process of making sure they add correctly.
- After students are comfortable with the game, you can start giving out only part of the deck, so that the total will not always be the same. For example, you can take out the 8s. Once students reach a total, discuss how the number should be different with the 8s gone.
- Another variation is to split a deck between two groups. Then, after both groups finish, add their totals together to check whether the addition was done correctly.

Activity 3: Making shapes on grid paper

In this activity, students create shapes and find the areas of those shapes in square units. They can begin with "counting the boxes" and move on to strategies that involve decompos-

ing the shapes and adding the areas. This activity may be used in grades 2–5.

Materials

- Lots of grid paper (half-inch grid size works well)
- Crayons or colored pencils
- Scissors (optional)
- · Plain paper and pencils for writing equations

Purpose

- Understand how addition is used in finding the area of rectangles and shapes built out of rectangles ("area is additive")
- Prepares students to understand how area can be used to model both addition and multiplication, as well as the DP

Steps

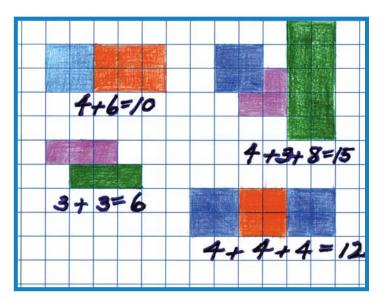
- 1. Students draw shapes on the grid paper. These can be as simple and basic or as creative and wild as the students like, as long as they follow the grid lines.
- 2. Students then color different regions of their shapes with different colors.
- 3. Students count up the area of each region, count up the area of the whole shape, and write an equation that expresses how the area of the whole shape is made up of the areas of the parts. Several examples are shown with activity 3 on page 307.

Variations and scaffolding

This activity can be adapted for grades 2–5. Here are some ways to change the level and mathematical focus:

- The teacher can provide the shapes at first, so that students can do the same problem in different ways and then compare solutions.
- Start with smaller shapes, so that students can easily count all the squares. This allows them to check their own work on the addition equation.
- As the students are ready, encourage them to be as creative as they wish. Many students will want to make designs, pictures, or complicated shapes.
- One variation is for the teacher to give the addition equation and ask students to draw a picture. "Who can draw a picture on the grid that represents 14 + 25 = 39?"
- Ask students to draw a large rectangle and divide it into two smaller rectangles. The equations they get this way lead directly to the DP.





Activity 4: Representing the whole multiplication table with arrays

Materials

- Grid paper, same size as in the previous activity
- Crayons or colored pencils
- Scissors

The basic activity

Students cut out a rectangular array for every multiplication fact from 1×1 up to 10×10 . These arrays are then labeled and pinned to a bulletin board in an organized way. The picture to the right shows the whole bulletin board, with a few of the 7s, 8s, and 9s not yet added.

Below is a close-up picture of a corner of the board.

This project can be done over several months, adding one or two arrays every day. Students can take turns making the arrays to add to the board.

Making connections with the distributive property

Note the orientation of the arrays. The fifth row, for example, is 5×1 , 5×2 , 5×3 , and so forth. All these arrays are five units high, and this makes them line up in a neat row.

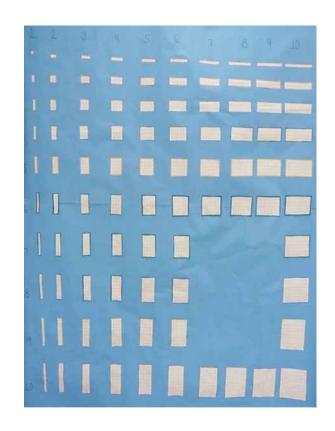
The fifth column is 1×5 , 2×5 , 3×5 , and so forth. All these are five units wide, and this makes them line up in a neat column.

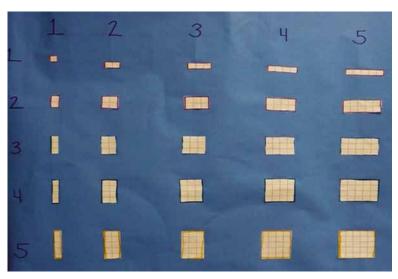
Also, you can clearly see in the fifth row that a 5×2 array next to a 5×3 array will give you a 5×5 array, and we can write this as the DP:

$$(5 \times 5) = (5 \times 2) + (5 \times 3)$$

Similarly, in the fifth column, we can see that

$$(5 \times 5) = (2 \times 5) + (3 \times 5).$$





Activity 5: Using the distributive property to understand multiplication

Materials

- · Lots of grid paper
- Crayons or colored pencils
- Scissors (optional)
- · Plain paper and pencils for figuring

Purpose

- Use the distributive property to decompose a harder multiplication problem into two easier ones.
- Model this by decomposing a rectangle into two smaller rectangles. This activity may be used in grades 3–5.

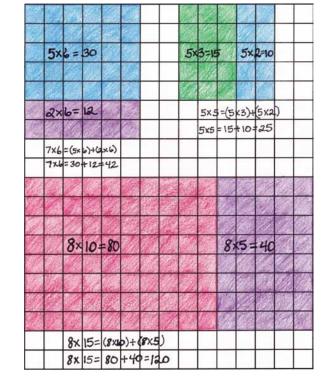
Steps

- 1. Students choose a multiplication problem that they want to solve or that they are having difficulty with. It can be one of the harder facts, like 7×6 , or a more complicated problem, such as 8×15 .
- **2.** The student draws the multiplication problem as an array or rectangle on the grid paper.
- **3.** The student divides the rectangle into two rectangles, and writes down the area for each one.
- **4.** The student writes two equations to show the relationship among the three multiplication problems.

Variations and scaffolding

The idea of decomposing a multiplication problem into smaller and easier problems starts with the simplest problems and leads eventually to understanding the standard algorithm for multiplying any two numbers.

- At first, the class can do simple examples together, like breaking apart 5×4 into 3×4 and 2×4 .
- Students who are working on memorizing their multiplication facts can choose one they are having trouble with and explore different ways to break it down into easier facts. For example, 7×6 can be broken down into 5×6 and 2×6 .



- When students are ready, they can start breaking rectangles apart into four smaller rectangles instead of two. For example, 12×25 can be decomposed into 10×20 , 10×5 , 2×20 , and 2×5 . This process leads eventually to an understanding of the standard algorithm.
- This same model can help students understand how to multiply fractions and mixed numbers. For example, $4 \times 6 \frac{1}{2}$ can be modeled as the area of a rectangle that is 4 units in one direction and 6 1/2 units in the other. The rectangle can be decomposed into a 4×6 rectangle and a $4 \times 1/2$ rectangle.